APPLICATION OF BAYES METHOD IN EVALUATION OF ROP/NOP TRIP SETPOINT - PHASE I

FINAL REPORT

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Disclaimer

EXECUTIVE SUMMARY

This document presents the results and recommendations from the work conducted under CNSC Research Project R612.1 “Application of Bayes method in evaluation of ROP/NOP trip setpoint – Phase I”.

A draft report was presented in a seminar at CNSC offices in Ottawa on 16 February 2016, and written comments were invited by 15 March 2016. I am pleased to note that the fundamental components of the report, namely the NOP equations, the two-level Monte Carlo simulation and the Bayesian modelling, were accepted with no critical comments.

RESULTS

Tasks 4.1 to 4.4 of the project have been completed.

Task 4.1. Review of the NOP/ROP problem - original general description in Ontario Hydro documentation and SIMBRASS theory manual to be provided by CNSC. This should be the starting point for developing the technical basis for the independent capability. Develop observations concerning the original statistical approach and method, shortcomings and limitations, and potential options for improvement.

The original statistical method, referred to herein as the ‘traditional’ method, has been found to have two important deficiencies.

1. The probability distributions used for sampling some of the uncertain quantities needed for evaluating the performance of a trip setpoint are not statistically sound.
2. Even if the distributions used were statistically sound, as a simple Monte Carlo solution the traditional method provides only an estimate of the frequency of successful trip, and cannot quantify the uncertainty around this estimate.

Options for improvement comprise using a two-level Monte Carlo algorithm that separates frequency from uncertainty, and using sound statistical methods for deriving probability distributions. These improvements are incorporated in the proposed statistical framework (Task 4.3).

Task 4.2. Starting from a CNSC staff paper describing the NOP "effectiveness" and "risk" problems and expected features and needs for the statistical framework, which will be finalized in the form of a "software specification" document based on the discussion at the start up meeting and feedback from the contractor and industry specialists from OPG and Bruce, provide expert opinion on: separation of uncertainties for NOP analysis and approaches for propagation of "ignorance" and "variability" candidate statistical approaches, other than Bayes method, which should be considered for implementation in the independent capability.

“Ignorance” and “variability” are formulated as E-uncertainty and A-uncertainty. A-uncertainty refers to the fact that the detailed state of the reactor when a slow loss of

regulation occurs, as defined by quantities such as channel powers, is variable over time and so cannot be known in advance. A-uncertainty about these quantities is described by probability distributions. E-uncertainty refers to the fact that these A-distributions are not known. E-uncertainty is described by probability distributions for parameters of the A-distributions.

It is important to separate these uncertainties, with the frequency of successful trip being defined through the A-uncertainties, and with uncertainty about this frequency defined by the E-uncertainties.

The principal statistical paradigms for quantifying these uncertainties are considered, with only the Bayesian paradigm being found practicable for the problem of evaluating the trip setpoint.

Task 4.3. Develop a high level technical description of a statistical framework (statistical method(s), assumptions and requirements for input data) for verification and confirmation of ROP/NOP trip setpoint values which can be used for both "effectiveness" and "risk" problems.

A Bayesian statistical framework is developed based on a two-level Monte Carlo algorithm.

The high level description defines sound statistical models for A-uncertainty and E-uncertainty applying to each of the groups of uncertain quantities, such as ripples or overpowers. Alternative models are provided with a view to assessing the sensitivity of outputs to each choice of model, and with guidance on identifying the most appropriate models.

The “effectiveness” and “risk” problems are encompassed in a general framework where frequencies are defined for the critical power ratio at trip to exceed different critical values, and probabilities are computed for these frequencies to reach different thresholds.

Task 4.4. Develop: a work plan for implementation of the proposed framework in a computer code, and generic criteria for manufacturing benchmark cases and testing.

A work plan is presented in the form of a “Software Appendix” to this document. This includes detailed specifications of inputs and outputs, as well as the algorithms to apply the statistical framework.

Various suggestions are made for validation and testing of the software, including face validity checks, benchmarking, used of reserved data and the creation of a log file.

RECOMMENDATIONS

A number of recommendations are made in the event that Phase 2 of this project is commissioned, involving the creation of the proposed software.

a. In addition to the ability to deliver quality assured scientific software, the team must include experts in Bayesian statistics, and particularly in Bayesian computation.
b. The team should be given some freedom to offer different models within the general structure of the statistical framework. It is in the nature of the NOP problem, and particularly in the fact that maximum and minimum operations feature in the equations, that the solution may be sensitive to details of the models. This is one reason why alternative models are already prescribed, but it also motivates encouraging a degree of innovation on the part of the development team.
c. The project should be considered as not simply delivering a product to specification but also involving a substantial statistical research element, and would require at least a six-month timescale.
d. The team should be able to call upon somebody such as myself, with understanding of both Bayesian statistical methods and the evaluation of NOP trip setpoints, to provide technical support and/or independent review, and to ensure that the user manual gives clear guidance to the CNSC on specifying input values and on interpreting the software's outputs.

Looking beyond the completion of Phase 2, the software will allow flexibility in the specification of code error variances and correlations, which can be used to explore the sensitivity of the outputs to changing the currently assumed, possibly simplistic, error structures. If the results are found to be sensitive to such changes, then it will be important to develop new research to quantify reliable and well-founded error structures, particularly examining spatial correlations. This would lie outside the scope of Phases 1 and 2 of the current project, and so would require a separately funded research project.

CONTENTS

INTRODUCTION

PROJECT WORK STATEMENT

The purpose of this document is to describe the results and findings from the work conducted under CNSC Research Project R612.1 “Application of Bayes method in evaluation of ROP/NOP trip setpoint – Phase I”.

The schedule of work for this contract specifies the following tasks.

Task 4.1. Review of the NOP/ROP[1] problem - original general description in Ontario Hydro documentation and SIMBRASS theory manual to be provided by CNSC. This should be the starting point for developing the technical basis for the independent capability. Develop observations concerning the original statistical approach and method, shortcomings and limitations, and potential options for improvement.

Task 4.2. Starting from a CNSC staff paper describing the NOP "effectiveness" and "risk" problems and expected features and needs for the statistical framework, which will be finalized in the form of a "software specification" document based on the discussion at the start up meeting and feedback from the contractor and industry specialists from OPG and Bruce, provide expert opinion on: separation of uncertainties for NOP analysis and approaches for propagation of "ignorance" and "variability" candidate statistical approaches, other than Bayes method, which should be considered for implementation in the independent capability

Task 4.3. Develop a high level technical description of a statistical framework (statistical method(s), assumptions and requirements for input data) for verification and confirmation of ROP/NOP trip setpoint values which can be used for both "effectiveness" and "risk" problems.

Task 4.4. Develop: a work plan for implementation of the proposed framework in a computer code, and generic criteria for manufacturing benchmark cases and testing

Task 4.5. Prepare a draft report and present the results in a seminar with CNSC staff and invited guests

Task 4.6. Finalize the report.

[1] ROP stands for “Regional Overpower Protection”. In this report, “NOP” will be used to include ROP.

CNSC staff provided the following additional definitions, expectations and clarifications in a white paper (as indicated in Task 4.2 description above) which was discussed at the start-up meeting and in follow-up discussions:

a. In the context of this project, the statistical framework (SF) is the computational framework for evaluating the frequency of failure of NOP system and is defined by the theory, mathematical model, algorithm, and the input information which implements the theory and mathematical model. As such, the statistical framework may include one or more computer programs, special models, and all other information needed to apply the computational framework, including:

   (1) procedures for treating the input and output information and related uncertainties;

   (2) specification of those portions of the analysis not included in the computer programs for which alternative approaches are used;

   (3) all other information needed to specify the computational procedure.

b. *Note:* Typically, the algorithm introduces new approximations, in addition to the approximations made in the theory and mathematical model, to either make the calculations tractable or to explicitly address issues related to the available input information (data), such as the input data uncertainty structure, or specifics of the application. It is therefore important (and expected) to identify and discuss all approximations and take them into account in the evaluation of the uncertainty in the predictions and extent of validity of the proposed statistical procedure.

c. In the context of this project, the main output parameter, or The Figure of Merit (FOM), is the frequency of failure to meet a postulated (input data) criterion for the minimum Critical Power Ratio at trip (CPR^{trip}).

d. The FOM should be calculated for two cases:

   1. *the case of failure to meet the design criterion which is avoidance of dry-out expressed as* $CPR^{trip} > 1.0$

   2. *the case of failure to meet a postulated surrogate for the safety criterion which is maintenance of fuel channel integrity expressed as* $CPR^{trip} > 1.0 + r$ *(where* r *is a positive quantity less than 0.1 and it is specified in input data)*

e. CNSC staff proposed the use of a Bayesian approach, because the NOP methodology is based on a probabilistic model and, by design, Bayesian methods natively consider the uncertainty associated with the parameters of a probability model (even if those uncertain parameters are believed to be fixed numbers) and are often recommended and used (as proven practice) in areas of reliability and probabilistic assessment as the proper way to make formal use of subjective information.

f. In the context of this project, the Bayesian SF is proposed to be used to estimate functional failure distributions for a NOP system with fixed design control variables (number of detectors, location of detectors and value of trip setpoint). Specifically, the Bayesian SF should answer the question: What is the estimated frequency of NOP system (functional) failure and the uncertainty around that estimate?

g. The distributions and quantities that result from the analyses would be the primary fodder for a decision making process, but the decision process itself is defined outside the Bayesian SF, i.e., the Bayesian SF is used to generate the input data for a decision-making analysis model. The decision-making analysis model is outside the objective and purpose of this project. Therefore, the Bayesian SF should be designed independently of any considerations about the decision-making analysis model.

h. In the context of this project, the model, mathematical formulation and algorithm will focus primarily on the option which uses the "superposition" approximation and, as such, allows the use of the same input data as used in OPG/BP NOP analysis. A more general option, which does not rely on the "superposition" principle and considers the possibility of more detailed and accurate input data, in particular the input data related to uncertainties in code predictions, should also be formulated.

i. The proposed Bayesian SF should include the capability to execute sensitivity analyses.

This document, with its Appendices, comprises the report of Task 4.5. It describes the objective, theory, model, algorithm, and key features of the proposed SF and an implementation work plan.

A draft report was presented in a seminar at CNSC offices in Ottawa on 16 February 2016, and written comments were invited by 15 March 2016. I am pleased to note that the fundamental components of the report, namely the NOP equations, the two-level Monte Carlo simulation and the Bayesian modelling, were accepted with no critical comments.

ORGANISATION

The main document is organised according to the list of tasks, with a section for each of Tasks 4.1 to 4.4. The work carried out and principal conclusions in each task are described. These conclusions are supported by detailed analyses and explanations in a Technical Appendix (TA).

A second appendix, the Software Appendix (SA), is a draft of a specification for the proposed computer code, intended as a comprehensive briefing for coding the proposed SF.

A third appendix lists the written comments made by Bruce Power and Ontario Power Generation following the 16 February seminar, with detailed dispositions.

TASK 4.1: REVIEW OF THE NOP/ROP PROBLEM

NOP METHODOLOGY

My review of the NOP/ROP problem inevitably began with reading several documents to understand the theory behind the NOP trip setpoint. Principal sources of information, as indicated in the contract work statement and CNSC staff white paper, were:

- A.L. Wight, Ontario Hydro Report No. 83254, September 1983
- "SIMBRASS 6.0 Software Theory Manual" (chapters related to design ("traditional") formulation: 3.1 to 3.4 and 3.11), NSS report G0127/RP/012, 2004

Additional sources of information were:

- M. Levine, P. Sermer, "CANDU Neutron Overpower Protection Systems Analysis Methodology Development – Part 1: Background", AMEC NSS Presentation at the startup meeting of Independent Technical Panel Review of OPG/BP Enhanced NOP Methodology, September 10, 2008
- Discussions with CNSC technical staff.

The principal components of the NOP methodology as used by OPG and BP may be outlined as follows.

- A Slow LOR is considered to begin with the establishment of a *perturbation* in the distribution of channel powers across the reactor, leading to a loss of regulation in which the reactor bulk power steadily increases.
- The NOP trip is designed to operate automatically to protect each individual channel against the possibility of *dry-out*, which will occur if the power in any channel becomes so high that it exceeds the *critical channel power* for that channel.
- A set of flux detectors, embedded in the core and organised into three *logic channels*, monitor flux in various discrete locations and implicitly monitor for an increase in bulk power. Formally, the NOP trip operates if at least one of the flux detector readings in each logic channel exceeds the *trip setpoint* (which is a fixed value, hard-wired into the reactor management system).
- The ratio of the critical channel power to the actual channel power for the limiting channel (the one for which this ratio is minimal) is called the margin to dry-out. In a Slow LOR, as bulk reactor power increases this ratio also decreases, and dry-out may occur if it reaches 1.
- The ratio of the trip setpoint to the reading of the limiting flux detector (which has the highest reading in its logic channel but is the lowest of the three logic channel maxima) is called the margin to trip. In a Slow LOR, as bulk power increases the margin to trip reduces, and trip occurs when it reaches 1. The trip setpoint is low enough to avoid dry-out if this happens before the margin to dry-out falls to 1.

- In order to choose an appropriate trip setpoint, or to assess whether a proposed trip setpoint is effective, the theory must relate the readings of the flux detectors to the channel powers. In doing so, the NOP equations introduce many quantities, but for a random future Slow LOR event the values of these quantities will not be known precisely.

Understanding of the NOP theory was important not only for Task 4.1 but also for Tasks 4.2, 4.3 and 4.4. In particular, it was necessary to find a formulation that was suitable for the SF of Task 4.3.

The theory in the sources quoted above also needed adapting to address the focus of this project as set out in the CNSC staff paper for Task 4.2, which was not on the derivation of a trip setpoint (as had been the focus of most of the sources) but on the chances of trip failing to prevent the possibility of dry-out, given a proposed trip setpoint value.

My expression of the NOP theory is given in the TA, section 2. In studying the NOP theory, I identified a possibility, which had not been mentioned explicitly in any of the sources I studied, where a perturbation is sufficiently large to pose a risk of dry-out even without any subsequent increase in bulk power. New equations were derived to address this case.

THE TRADITIONAL APPROACH

The original, "traditional" approach to choosing a trip setpoint is described in the "SIMBRASS 6.0 Software Theory Manual", primarily in sections 3.2.1 to 3.2.3.

1. Probability distributions are assigned to all of the uncertain quantities in the NOP equations.
2. A random draw is made from each probability distribution, to produce a value for every quantity in the equations.
3. Using the NOP equations, the *required trip setpoint* (RTSP) is computed as the trip setpoint such that the margin to trip equals the margin to dry-out. Thus, the RTSP is the value such that as bulk reactor power increases during a Slow LOR the trip will occur *just in time* to prevent the risk of dry-out.
4. Steps 2 and 3 are repeated many times to produce a large Monte Carlo sample of RTSP values.
5. The chosen trip setpoint is a value such that a suitably high proportion (e.g. 98%) of the sampled RTSP values exceed that chosen value.

The focus of the present project is to assess the efficacy of a proposed trip setpoint, and the traditional approach might be adapted for this purpose: the efficacy is given by the proportion of sampled RTSP values that are greater than the proposed trip setpoint value. This is a Monte Carlo estimate of the probability that the NOP trip will operate to prevent dry-out in a random Slow LOR event.

If the probability computed in step 5 is sufficiently high, e.g. 95% or 98%, the proposed trip setpoint can be viewed as effective[2].

Part of Task 4.1 is to develop observations regarding this method, and to identify "shortcomings and limitations, and potential options for improvement".

Shortcomings and limitations. In my judgement, the traditional method suffers from two deficiencies.

A. The way that probability distributions are assigned to some of the uncertain quantities in the NOP equations is not statistically sound. This deficiency is explained in detail in the TA, section 4.2.5.
B. The probability that is calculated in this method is not a suitable basis for judging the efficacy of a proposed trip setpoint. This deficiency is demonstrated in simple terms at the end of the section on Task 4.2 below.

Options for improvement. Improvements should ideally address both of the above shortcomings. Such options were considered as part of Task 4.2 and are discussed in the relevant section below. The principal finding of Task 4.2 is to propose a Bayesian SF in which the Monte Carlo approach of the traditional method is retained but adapted to compute more suitable expressions of efficacy, addressing deficiency B. The Bayesian approach also provides statistically sound constructions of the appropriate distributions for all the uncertain quantities, addressing deficiency A.

[2] For the purposes of the discussion here, we focus on measuring efficacy in terms of the chance that the NOP trip will operate successfully to prevent dry-out. Other measures of efficacy are introduced at the end of the section on Task 4.2.

TASK 4.2: STATISTICAL APPROACHES

TYPES OF UNCERTAINTY

Part of Task 4.2 is to determine whether, and if so how, in order to assess the efficacy of a proposed trip setpoint there should be a separation of uncertainties, so that different types of uncertainty are handled separately.

A distinction is often made between two types of uncertainty, namely *aleatory* and *epistemic* uncertainties[3]. Aleatory uncertainty applies to things that vary randomly, whereas epistemic uncertainty applies to something that may have a unique, fixed value but we do not know that value. The distinction can be subtle and is not always well-defined. However, the following example will clarify the distinction that will be important for this project.

Consider, for instance, the channel power CP_i in fuelling channel i at the time when a particular perturbation is initiated. This is uncertain for two reasons. First, we imagine this perturbation being initiated at some random time point, so there is uncertainty about what state the reactor will be in at that time. This is aleatory uncertainty, because we are imagining a random future time point and hence the fuelling state is random. If we knew the reactor state (which we can define as including all relevant details such as the fuelling history, the disposition of reactor management devices and drift in calibration of the flux detectors), then in principle CP_i has a unique, fixed value. We could compute this value using the SORO computer code, but this does not give the true value of CP_i because no computer code is perfect. Errors in the computation, whether due to rounding error, imperfect solution of the systems of differential equations, or imperfect science, mean that the computed value will be different from the true value. This is epistemic uncertainty – there is a true value but we do not know it exactly.

I will refer to uncertainties arising from the random *actual* fuelling state when a perturbation is initiated as A-uncertainties, and uncertainties arising from imperfect *evidence* as E-uncertainties. Most of the quantities in the NOP equations are subject to both A-uncertainty and E-uncertainty. Like CP_i, most quantities take different values at different times, depending on the reactor state, and so are subject to A-uncertainty. Given the reactor state, these quantities would in principle have unique, fixed values but these true values are unknown. They may be computable or estimable, but there will always be errors in such computations or estimations, and so most quantities are subject also to E-uncertainty.

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[3] Other terms are frequently used, with the result that there is often confusion about the distinction between the two types of uncertainty. Aleatory uncertainty is sometimes referred to as variability or randomness or irreducible uncertainty. Epistemic uncertainty is sometimes called just uncertainty (as opposed to variability or randomness), knowledge uncertainty or reducible uncertainty. My choice of the terms A-uncertainty and E-uncertainty are intended to avoid philosophical or linguistic debate, and are defined specifically for the NOP context.

Another important component of E-uncertainty is that we do not know how often the reactor will be in different states in the future. In general, we can characterise A- and E-uncertainties as follows. A quantity like CP_i will vary randomly from time to time, and this variability will be expressed in a probability distribution – this is A-uncertainty. Its A-distribution specifies what values of the quantity are possible, and how probable different values are. But some aspects of that probability distribution (the possible values of the quantity and/or the probabilities) are uncertain – this is E-uncertainty.

Associated with the two types of uncertainty are two principal philosophies of statistical analysis. These are discussed in the TA, section 4.2.

SEPARATION OF UNCERTAINTIES

For a given proposed trip setpoint, then depending on the reactor state at the time when a particular perturbation initiates a Slow LOR, the NOP trip *may or may not* operate *successfully* (i.e. before the reactor bulk power increases to the point where the power in one or more channels reaches its critical channel power and there is a risk of dry-out). An effective proposed trip setpoint will have a high frequency of success.

One way to evaluate that frequency of success would be to initiate a perturbation at many random times and count how often the NOP trip operates successfully. Of course, that is totally impractical, but suppose we had no E-uncertainty. Then we would know completely the probability distributions of all the quantities in the NOP equations. In principle we could now use the NOP equations to derive the probability of success, in the same way as we can derive the probability that $X + Y > 0$ once we know the probability distributions of X and Y. In practice, the NOP equations are much more complex than "$X + Y > 0$", and the mathematical analysis to perform this derivation would be extremely demanding. A simple alternative would be to use the Monte Carlo method – we could sample future values of all these quantities and apply the equations to determine whether the trip would operate successfully, then evaluate the probability of success by counting the frequency of successes in a large number of simulated future reactor states.

Notice that the frequency of success is defined entirely with respect to the A-uncertainties, assuming no E-uncertainty. Because the quantities are also subject to E-uncertainty, we do not know what that frequency of success is – formally, it is also subject to E-uncertainty. For instance, we might wish the trip to operate successfully at least 98% of the time, but we could not be sure whether the true frequency of success was 98% or more.

Assessment of the efficacy of a proposed trip setpoint necessarily involves a separation of uncertainties. We need to assess

a) how sure we are, with respect to the E-uncertainties, that
b) the frequency of success, with respect to the A-uncertainties,

is at least as high as the desired threshold.

The way that "how sure we are" is defined and expressed depends on the statistical approach adopted.

STATISTICAL APPROACHES

A statistical analysis to quantify "how sure we are" requires three steps.

1. For each quantity that is subject to A-uncertainty, a suitable model is assumed for its A-distribution. The model will typically have one or more *parameters*, and by assuming a model the E-uncertainty is focused on uncertainty regarding the values of the parameters. For example, the A-distribution of flux detector drift may be assumed to be normal with zero mean and variance v. Then E-uncertainty regarding detector drift resides in the parameter v.
2. The data that comprise the evidence regarding the parameters are modelled. For example, there might be a sample of observations of detector drift, which are modelled as independent, also normally distributed with zero mean and variance v.[4]
3. The data and the models in step 2 are used to derive statistical inference about the parameters. Then using the models in step 1 and the NOP equations, inference is derived to express "how sure we are" about the frequency of success.

For step 3, there are two principal statistical inference paradigms, known as *frequentist* and *Bayesian* inference. They differ quite fundamentally in how they handle epistemic uncertainties.[5]

Bayesian methods express epistemic uncertainties using probabilities. Therefore, all the E-uncertainties will be represented by probability distributions. Bayesian inference in step 3 leads to E-distributions for the parameters, and eventually to a *probability distribution* for the frequency of success. Using this distribution, we can quantify precisely "how sure we are" – it is simply the probability that the frequency equals or exceeds the desired threshold.

Frequentist inference does not allow epistemic uncertainties to have probability distributions. The formulation of "how sure we are" is consequently more oblique. The result of a frequentist analysis for the NOP problem will be a *confidence interval* for the frequency of success. It might for instance state that with 90% confidence the frequency of success lies between 94% and 99%. A confidence interval may usually be adapted to compute a degree of confidence that the frequency of success equals or exceeds a given threshold.

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[4] The models in steps 1 and 2 are the same here because we are supposing data comprising direct observations of the quantity. Although the same model is used, it has different purposes. In step 2 its role is to allow us to learn about v from past observations, whereas in step 1 it describes a detector drift at some future time. The models in the two steps can be quite different, depending on the evidence for the parameters.

[5] These two are the dominant philosophies in Statistics. The TA, sections 4.2.2 and 4.2.3, gives some consideration also to fiducial inference and p-boxes.

Confidence is not the same thing as probability. Differences between the two principal inference paradigms are explored in more depth in the TA, section 4.2, but in general either form of inference may be used. In the field of Statistics there are passionate advocates on each side – Bayesian statisticians who regard frequentist methods as unsound and frequentist statisticians who believe Bayesian methods are unscientific – but there are others who consider that either approach may be used in practice, the choice being a matter of taste or convenience.

However, it is my opinion that a frequentist implementation of step 3 in this case would be extremely difficult. It would take a very substantial research programme to develop such a solution, with no assurance of eventual success.

A Bayesian solution will also not be straightforward, but is nevertheless in my judgement enormously more practical than a frequentist approach. A key to this opinion is the availability of powerful computational tools. A Bayesian method can compute E-distributions for the parameters for each A-uncertainty model separately using Markov chain Monte Carlo (MCMC) techniques that are not available to frequentist methods. These can then be propagated through the NOP equations using two-level Monte Carlo, which again does not seem possible in the frequentist approach because it does not allow distributions for the E-uncertainties.

The SF developed under Task 4.3 is therefore a Bayesian framework.

FIGURES OF MERIT

A draft CNSC staff white paper describing the NOP "effectiveness" and "risk" problems was presented at the start-up meeting for this project in Ottawa. In order to formulate these problems, we define the *critical power ratio at trip* (CPR^{trip}) to be the value of the margin to dry-out at the moment when the NOP trip is triggered by the establishment of a perturbation with a subsequent increase in reactor bulk power. We have hitherto been supposing that the required measure of the efficacy of a proposed trip setpoint is the frequency of success, where success can now be simply expressed as $CPR^{trip} \geq 1$.

More generally, we can define

$$Q(C) = \text{Freq}_A\ (CPR^{trip} \geq C)\ ,$$

where Freq_A denotes frequency with respect to the A-uncertainties. The frequency of success then becomes $Q(1)$. For any given threshold T, the Bayesian solution computes

$$P(C, T) = \text{Prob}_E\ (Q(C) \geq T)\ ,$$

where Prob_E denotes probability with respect to the E-uncertainties.

An important difference between the “effectiveness” and “risk” problems in the CNSC staff paper lies in the treatment of the perturbation. A finite, discrete set of perturbations are to be considered. In the effectiveness problem, we consider only Slow LOR arising with a

particular perturbation. If $Q_J(C)$ denotes the frequency with which $CPR^{trip} \geq c$ when we restrict attention to LOR arising with perturbation J, then the appropriate *Figure of Merit* (FoM) for the effectiveness problem is

$$P_J(C, T) = \text{Prob}_E\,(Q_J(C) \geq T)\ ,$$

computed for each perturbation J.

In the risk problem, given that a LOR arises with one of the perturbations in the normal design set, these perturbations are supposed to occur with A-probabilities p_J, and the FoM $P(C, T)$ can now be written

$$P(C, T) = \sum p_J \times P_J(C, T)\ ,$$

where the summation is over all possible perturbations.

THE DEFICIENCIES OF THE TRADITIONAL APPROACH

The "Traditional" method was described in Task 4.1 above, where two deficiencies, named A and B, were identified. We now return to this topic in the light of the preceding discussion.

First, note that the traditional method places probability distributions on all uncertainties. In terms of the dominant statistical inference paradigms, this cannot be a frequentist method because it is giving probability distributions to epistemic uncertainties. It is therefore implicitly Bayesian. However, the essence of deficiency A of the traditional method is that these distributions are not correctly derived according to Bayesian probability theory. [6]

The traditional method does not make a separation between A- and E-uncertainties. A single Monte Carlo simulation is performed to compute a frequency of success. If we ignore the first deficiency, the second deficiency is that this frequency is the *expectation* (with respect to the E-uncertainties) of $Q(1)$. It is therefore an estimate of the frequency of success but without any expression of "how sure we are" that the true frequency will be above this estimate or close to this estimate.

[6] In the TA, section 4.2.2, the derivations are seen to be at least partly in accordance with fiducial inference.

TASK 4.3: STATISTICAL FRAMEWORK

Because of the many different uncertain quantities, the variety of forms of available data and the complex structure of some of the quantities, the Statistical Framework is complex. Full details are given in the TA, section 5, and in the SA, section 7.

STEP 1: A-UNCERTAINTY MODELLING

Many aspects of the reactor state will vary from time to time, and therefore give rise to A-uncertainty when a Slow LOR is initiated at a random future time, including the following.

- The fuelling state, which is a combination of the history of refuelling actions and the current disposition of reactor management devices. A-uncertainty is induced in the shape[7] of channel powers and of the flux levels at the flux detector locations, both before and after a given perturbation is established. In principle, A-uncertainty also affects the critical channel powers for a given perturbation.
- The total reactor power at the time when the perturbation is initiated (and before any subsequent bulk power increase). The actual channel powers and flux levels are a product of their shape and the reactor power, so this contributes further A-uncertainty to those quantities.
- Flux detector drift. The drift in each flux detector since it was last calibrated contributes A-uncertainty to the actual detector readings at the time when the perturbation is initiated. (The drift is assumed to remain constant over subsequent increase in bulk power.)

The SF proposes models for these A-uncertainty distributions. The fuelling state plays a vital role in the computation of CPR^{trip} by controlling the distribution of channel powers, and so the modelling of A-distributions arising from fuelling state is very important. Accordingly, a variety of models is proposed in the SF, and the software will allow different models to be used in order to assess sensitivity of the Figures of Merit to modelling assumptions.

The simplest model is to assume that the fuelling state at a random future time will be the same as a randomly selected one of the states for which we have SORO data (see the discussion of Step 2 below). Although this is intrinsically an unrealistic assumption, we have data for a relatively large number of fuelling states and it is not unreasonable to suppose that these cover the range of possibility reasonably well.

A second model assumes that the shape of channel powers and flux levels at a future time are drawn from a multivariate normal distribution. Since this may also be unrealistically restrictive, a third model assumes that the underlying A-distribution is a mixture of multivariate normal distributions.

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[7] The more common word here is the *distribution* of channel powers, but we use the word "shape" to avoid confusion with probability distributions.

STEP 2: DATA MODELLING

The principal data consist of outputs from the computer codes SORO, RFSP and TUF.

SORO outputs are available for a relatively large sample of fuelling states with no perturbation. For each sampled fuelling state, the SORO outputs are estimates of the *ripples*, i.e. the ratios of channel powers for each fuel channel, scaled to sum to the design reactor power, divided by reference channel powers. The data model says that these are equal to the true ripples for those fuelling states plus code errors that have a multivariate normal distribution.[8] The variance matrix is assumed known, based on the known accuracy of the SORO code. Where A-distributions are assumed for the true powers, rather than assuming that the only possible fuelling states are those for which we have data, the data are modelled as arising from a random sample of fuelling states.

Two forms of RFSP data may be provided to the software. The more extensive data comprise RFSP estimates of overpowers (channel overpowers and flux overpowers) for each of the fuelling states where SORO data are available, combined with each of the possible perturbations. For each given perturbation, we have sets of RFSP overpowers that correspond to the sets of SORO ripples. We assume the same form of data model, but with a variance matrix reflecting the known accuracy of the RFSP code.

The other form of RFSP data is a set of estimated overpowers for each perturbation, computed from the time-averaged fuelling state. The data model assumes again that the computed values are equal to the true values plus multivariate normal errors, with variance matrix reflecting the known accuracy of RFSP.

TUF data provide critical channel powers for each fuel channel. As with the RFSP data, these may be provided for each combination of perturbation and sampled fuelling state, or else for each perturbation with a time-averaged fuelling state. The data models are as for RFSP data, but with variance matrix appropriate to the known accuracy of TUF.

STEP 3: INFERENCE

Step 3 in the Bayesian SF begins with formulating prior distributions for the parameters. These are standard weakly informative prior distributions.

Then the Bayesian statistical paradigm is invoked to combine the data with the prior distributions, using the data models, to produce posterior distributions. These are the E-distributions for the parameters. In most cases, the analysis is too complex to derive posterior distributions analytically, so the software will employ appropriate forms of MCMC

[8] Strictly, for all three computer codes it is the logarithms of their outputs that are assumed equal to the logarithms of the corresponding true values plus multivariate normal errors. This is more natural, statistically, than assuming additive errors on the original scale.

computation to derive the E-distributions. MCMC produces a sample from the posterior distribution, rather than a mathematical form for the distribution, but this is perfect for the final part of step 3.

That final part is a Monte Carlo simulation to derive the E-distribution of $Q(C)$ for any desired value of C, and thereby to evaluate $P_J(C, T)$ and $P(C, T)$ for the desired value of T. This is a two-level simulation, reflecting the separation of A- and E-uncertainties, as follows.

1. Repeat for each perturbation J {

    Set counter $p_J(C, T) = 0$

    Repeat M1 times (outer loop) {

        Randomly sample values of parameters from their E-distributions

        Set counter $q(C)=0$

        Repeat M2 times (inner loop) {

            Randomly sample values of quantities from their A-distributions

            Compute CPR^{trip}

            Increment $q(c)$ if $CPR^{trip} \geq C$ }

        Set $Q(C) = q(C)/\text{M2}$

        Increment $p_J(C, T)$ if $Q(C) \geq t$ }

    Set $P_J(c,t) = p_J(c,t)/\text{M1}$ }

2. Compute weighted average $P(C, T) = \sum_J p_J * P_J(C, T)$

The simulation sizes M1 and M2 should be large enough to produce stable Monte Carlo estimates. Notice that in the inner loop the A-distributions are fixed using the values of parameters that have been sampled in the outer loop.

TASK 4.4: WORKPLAN

The Software Appendix, SA, contains all the technical material that should be in a specification for the software package to implement the SF.

TESTING AND BENCHMARKING

Like any major software development, the NOP trip setpoint assessment software must be thoroughly tested and quality assured, underpinned by the principles of V&V (verification and validation). Verification is the process of checking that the software is a correct implementation of the mathematical and logical structure of the required solution, and is done by checking that outputs are correct for given inputs. Usually, the correct values are only known for relatively simple cases, and so verification also relies on checking *face validity*, i.e. ensuring that as the inputs change in particular ways the outputs change as expected.

For instance:

- If the quantity of input data (SORO and/or RFSP datasets) is reduced, the overall E-uncertainty should increase, and hence the E-uncertainty in CPR^{trip} should increase.
- If the specified variances of code errors are increased, then again the E-uncertainty in CPR^{trip} should increase.
- Perturbations that have uniformly smaller effects on channel powers should lead to generally higher values of CPR^{trip}, and in particular should have higher $P_J(C, T)$.

The validation process is a check on the solution. Thus, if the software has passed verification testing and can therefore be assumed to be a correct implementation of the solution, we ask: Is the solution fit for purpose? To some extent, the above face validity tests address this question, because a failure of such a test may not imply that the software has implemented the solution wrongly; it may be that the software is a correct implementation but the solution is itself so poor that it does not have face validity.

Another tool for validation is benchmark testing. Test examples are constructed where the correct solution is known, and the software solution is checked to see if it is close enough to the correct solution to be fit for purpose. There is no real data for which true values are known, so it is necessary to fabricate “realistic” data.

For instance:

- The datasets of SORO, RFSP and TUF data are assumed to be true values for a sample of fuelling states. The software is first run using these data with no E-uncertainty (e.g. zero code error variances) to obtain the true $Q(C)$ value. Then new data are generated by taking these “true values” (or by creating new sets of true values from mixtures of the original data), and then adding random code errors consistent with the known accuracies of the codes. The software is run with several new

datasets generated in this way, to check that the outputs are consistent with the true $Q(C)$.

Another useful validation test uses held-back data. The software is run with a reduced set of data, for instance two-thirds of the original number of fuelling state samples, to predict the values of the remaining, held-back, data. This requires the inner loop to be modified so that instead of computing CPR^{trip} it simulates datasets from additional random fuelling states. The inner loop normally simulates true vales, but it is now modified to add random code errors to simulate computed data. In this way a sample of computed datasets from their predictive distribution is generated, and this is compared with the held-back data.

RECOMMENDATIONS

If the CNSC chooses to commission the second phase of this project, to produce the software package according to the SF, then I make the following recommendations.

Two separate kinds of expertise will be needed. Naturally, the development team must have experience in the delivery of quality assured scientific software. Second, though, they must include experts in Bayesian statistics, and particularly in Bayesian computation. In Canada, the necessary expertise is likely to be available in just a few university statistics groups, perhaps only in UBC, SFU and McGill. There may be some value in also advertising outside Canada.

The team should be given some freedom to offer different models within the general structure of the SF. It is in the nature of the NOP problem, and particularly in the fact that maximum and minimum operations feature in the equations, that the solution may be sensitive to details of the models. This is one reason why alternative models are already prescribed, but it also motivates encouraging a degree of innovation on the part of the development team.

The project should be considered as not simply delivering a product to specification but also involving a substantial statistical research element – towards devising reliable and efficient algorithms for the Bayesian computations, towards statistical critique of the models proposed in the SF and, where appropriate, towards developing alternative models. It seems to me that it would require at least a six-month timescale.

Alongside commissioning a team to develop the software in phase 2, the CNSC may find it useful to commission me for an additional package of work. This would include providing technical support and/or independent review. Furthermore, I recommend that a user manual is written to accompany the contractors' technical manual for the software, giving guidance to the CNSC on specifying input values and on interpreting the software's outputs, and this might form a part of this additional work package.

The software will allow flexibility in the specification of code error variances and correlations, which can be used to explore the sensitivity of the outputs to changing the currently assumed, possibly simplistic, error structures. If the results are found to be

sensitive to such changes (and I judge this to be likely), then it will be important to develop new research to quantify reliable and well-founded error structures, particularly examining spatial correlations. This would lie outside the scope of phases 1 and 2 of the current project, and so would require a separately funded research project.

APPENDICES

The following Appendices are provided as separate documents.

APPENDIX A. TECHNICAL APPENDIX

Document “R612.1 - Appendix A - Technical.pdf”

APPENDIX B. SOFTWARE APPENDIX

Document “R612.1 - Appendix B - Software.pdf”

APPENDIX C. DISPOSITION OF COMMENTS

Document “R612.1 - Appendix C - Dispositions.docx”

APPLICATION OF BAYES METHOD IN EVALUATION OF ROP/NOP TRIP SETPOINT — PHASE I
Technical Appendix

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March 23, 2016

1 Background

This document provides a variety of technical details to accompany the main report, "Application of Bayes Method in Evaluation of ROP/NOP Trip Setpoint — Phase I", and the Software Appendix (SA) to that report.

2 NOP Methodology

This section introduces notation and develops the principal equations that form a part of the SA.

2.1 Channel powers, fuelling states, perturbations and loss of regulation

For the purposes of this work, the status of a CANDU reactor at any time may be summarised by the power being produced in each of its hundreds of fuel channels. At time t, we denote the **channel power** in channel i (where subscript i ranges from 1 to Ni, the number of channels in the reactor) by CP_i^t. The sum of channel powers at time t is the bulk **reactor power** $RP^t = \sum_i CP_i^t$. Channel powers and reactor power are measured in kilowatts (kW).

In this work we are specifically concerned with the condition known as slow **loss of regulation** (LOR), in which an unplanned **perturbation** of channel powers (or 'flux shape') occurs and leads to a loss of regulation of the reactor in which the reactor power then increases steadily.

In order to formulate LOR effects, we will use the following notations.

- $CP_i^{n(k)}$ is the channel power in channel i under *normal* (i.e. unperturbed) operation with **fuelling state** k. The term 'fuelling state' is a shorthand

for not only the status of the reactor as a result of the history of fuelling operations to the present time, but also the normal management of the reactor through the regulating devices.

- $CP_i^{p(J,k)}$ is the channel power in channel i after *perturbation* J has arisen when the reactor was in fuelling state k, but before any subsequent increase of reactor power.

During LOR, it is assumed that first the perturbation is established with no change in bulk reactor power, so that

$$\sum_i CP_i^{p(J,k)} = \sum_i CP_i^{n(k)} = RP^{n(k)} \ , \tag{1}$$

and then the channel powers increase uniformly. Hence, if at time t there is a LOR in which perturbation J has arisen when the reactor was in fuelling state k and regulation loss has led to an increase in reactor power by factor $F > 1$, we have

$$CP_i^t = F \times CP_i^{p(J,k)} \ . \tag{2}$$

2.2 Ripples and channel overpowers

When the reactor is designed and commissioned, the **design reactor power** DRP is established, together with **reference channel powers** CP_i^{ref} such that $DRP = \sum_i CP_i^{ref}$.

It is usual to refer the channel powers at any time to reference channel powers through the equations:

$$CP_i^{n(k)} = PF^{n(k)} \times R_i^{n(k)} \times CP_i^{ref} \ , \tag{3}$$

$$CP_i^{p(J,k)} = COP_i^{p(J,k)} \times CP_i^{n(k)} \ , \tag{4}$$

where the **power factor** $PF^{n(k)} = RP^{n(k)}/DRP$ relates the reactor power in fuelling state k to the design reactor power, the $R_i^{n(k)}$ are referred to as the **ripples** associated with fuelling state k, and the $COP_i^{p(J,k)}$ are referred to as the **channel overpowers** associated with perturbation J arising when the reactor was in fuelling state k. These quantities are dimensionless.

2.3 Critical channel powers and margin to dryout

If the power at any point in the reactor becomes sufficiently high, there is a risk of **dryout**, where a spot occurs on the fuel element sheath that is inadequately cooled and so can lead to rupture, with potentially unwanted consequences.

For a given perturbation J and fuelling state k, we can identify **critical channel powers** $CCP_i^{p(J,k)}$, such that if the power in channel i exceeds

$CCP_i^{p(J,k)}$ after perturbation J when the reactor was in fuelling state k then dryout may occur in that channel.

If at time t there is a LOR in which perturbation J has arisen when the reactor was in fuelling state k and regulation loss has led to an increase F in reactor power the ratio

$$CPR_i^t = \frac{CCP_i^t}{CP_i^t} = \frac{CCP_i^{p(J,k)}}{F \times COP_i^{p(J,k)} \times CP_i^{n(k)}}$$

indicates whether there is a risk of dryout in channel i. If $CPR_i^t \geq 1$ then dryout will not have occurred in this channel, and the margin by which CPR_i^t exceeds 1 gives a safety factor. In fact, setting $F = 1$, the ratio $CCP_i^{p(J,k)}/CP_i^{p(J,k)}$ is the **margin to dryout** in channel i when perturbation J has arisen when the reactor was in fuelling state k, before any rise in overall reactor power. The minimum of CPR_i^t over all channels will be referred to as the **critical power ratio**

$$CPR^t = \min_i CPR_i^t = \min_i \frac{CCP_i^{p(J,k)}}{F \times PF^{n(k)} \times COP_i^{p(J,k)} \times R_i^{n(k)} \times CP_i^{ref}} \ . \quad (5)$$

2.4 Flux detectors, calibration and detector drift

In order to monitor for the possibility of LOR, a number of **flux detectors** are situated at points in the core to measure the flux (and so effectively the power) at those points. The detectors are organised into three **logic channels**, which are quite different from fuel channels. The detectors in a given logic channel are located at different points along the length of various fuel channels dispersed around the core.

The detectors are regularly **calibrated**, so that they can be sensitive to changes in reactor power. The calibration depends on the current fuelling state and reactor power, specifically, every detector is set to read $CPPF^{cal} \times IRP^{cal}$. The dimensionless **channel power peaking factor** $CPPF^{cal}$ depends on the fuelling state and IRP^{cal} is the **indicated reactor power**, at the time of calibration. Indicated reactor powers are expressed relative to DRP and are therefore also dimensionless.

The definition of $CPPF^{cal}$ is

$$CPPF^{cal} = \max\left\{CPPFm, MR^{cal}\right\} \ , \quad (6)$$

where $CPPFm$ is a predefined constant and MR^{cal} is a value based on the fuelling state[1] at, and leading up to, the calibration time.

The **detector reading** at time t from flux detector d in logic channel L is denoted by $DR_{L,d}^t$. If at time t there is a LOR in which perturbation J

[1] The notation here reflects the fact that MR^{cal} is a maximum ripple, over a central region of the core, in SORO calculations at and/or leading up to the calibration time.

has arisen when the reactor was in fuelling state k at the last calibration and regulation loss has led to an increase F in reactor power,

$$DR^t_{L,d} = F \times DD_{L,d} \times FOP^{p(J,k)}_{L,d} \times CPPF^{cal} \times IRP^{cal} \ , \tag{7}$$

where the dimensionless $FOP^{p(J,k)}_{L,d}$ are referred to as the **flux overpowers** associated with perturbation J arising when the reactor was in fuelling state k. The additional dimensionless factor $DD_{L,d}$ is the **detector drift** in detector d in logic channel L at the time of the perturbation, and accounts for detector errors, errors in physically calibrating the sensors and small changes in conditions over time since the last calibration.

2.5 The trip setpoint

The flux detectors protect against LOR and dryout through a **trip setpoint** TSP that is preset and hard-wired into the reactor control systems. The NOP protective action will trip if a **trip condition** is satisfied under which some detector readings reach TSP.

For the purposes of this project, the trip condition is that at least one detector reading reaches TSP in each of the logic channels. Hence

$$TSP = \min_L \max_d DR^{trip}_{L,d} \ , \tag{8}$$

where

$$DR^{trip}_{L,d} = F^{trip} \times DD_{L,d} \times FOP^{p(J,k)}_{L,d} \times CPPF^{cal} \times IRP^{cal} \tag{9}$$

and F^{trip} is the increase of reactor power at the time when the trip occurs. We can solve equations (9) and (8) to obtain

$$F^{trip} = \frac{TSP}{\min_L \max_d \left(DD_{L,d} \times FOP^{p(J,k)}_{L,d} \right)} \times \frac{1}{CPPF^{cal} \times IRP^{cal}} \ . \tag{10}$$

2.6 Critical power ratio at trip

The reactor will trip successfully, i.e. before dryout can occur, if the margin to dryout for every channel is at least 1 when F equals F^{trip}. Accordingly we define the **critical power ratio at trip** to be

$$CPR^{trip} = \min_i \frac{CCP^{p(J,k)}_i}{F^{trip} \times CP^{p(J,k)}_i} \ .$$

Substituting (10), we have

$$\begin{aligned} CPR^{trip} = \min_i & \frac{CCP^{p(J,k)}_i}{PF^{n(k)} \times COP^{p(J,k)}_i \times R^{n(k)}_i \times CP^{ref}_i} \\ & \times \min_L \max_d \left(DD_{L,d} \times FOP^{p(J,k)}_{L,d} \right) \times \frac{CPPF^{cal} \times IRP^{cal}}{TSP} \ . \end{aligned} \tag{11}$$

Then the trip operates successfully to avoid dryout if $CPR^{trip} \geq 1$.

Calculation of CPR^{trip} is a fundamental task in this project.

2.7 Trip during perturbation

It is possible, in practice if the flux shape represents a large perturbation of channel powers, for equation (10) to give a F^{trip} value less than 1. In this case, trip will occur before the flux shape has become fully established, and not due to a subsequent rise in reactor power. If we simply wish to evaluate whether the trip occurs successfully to prevent dryout, we can first check whether CPR^{trip} will still be at least 1 once the flux shape is fully established. The relevant formula now is

$$CPR^{cal} = \min_i \frac{CCP_i^{p(J,k)}}{CP_i^{p(J,k)}} \ . \tag{12}$$

If $F^{trip} < 1$ but $CPR^{cal} \geq 1$ then the trip will occur before dryout can occur.

If $CPR^{cal} < 1$, and in general in order to compute CPR^{trip}, we need to make an assumption about how the flux shape develops. We introduce a parameter E to model the development of the flux shape, which varies from $E = 0$ at the initiation of the perturbation to $E = 1$ when the flux shape is fully established. Now the detector reading at trip for detector d in logic channel L is,

$$DR_{L,d}^{trip} = DD_{L,d} \times (FOP_{L,d}^{p(J,k)})^{E^{trip}} \times CPPF^{cal} \times IRP^{cal} \ , \tag{13}$$

where E^{trip} is the value of E when the trip occurs. E^{trip} is the solution of equations (8) and (13), although now it is not possible to solve the equations explicitly.

The critical power ratio at trip now becomes

$$CPR^{trip} = \min_i \frac{CCP_i^{p(J,k)}}{PF^{n(k)} \times (COP_i^{p(J,k)})^{E^{trip}} \times R_i^{n(k)} \times CP_i^{ref}} \ . \tag{14}$$

Theoretically, the solution of equations (8) and (13) could yield a negative value for E^{trip}, but this should not arise because it would mean that a trip would occur during normal operation of the reactor without any unplanned perturbation.

2.8 Superposition and the CCP approximation

Two approximations are often used to simplify the NOP computations.

The first approximation concerns the channel powers and flux levels. Although in general the effect of the onset of a particular perturbation can be complex, and in particular the channel overpowers and flux overpowers will depend on both the perturbation J and the fuelling state k, a simplifying assumption known as the **superposition principle** asserts that we can write

$$COP_i^{p(J,k)} =_S COP_i^{\{J\}} \ , \qquad FOP_{L,d}^{p(J,k)} =_S FOP_{L,d}^{\{J\}} \ , \tag{15}$$

where $COP_i^{\{J\}}$ and $FOP_{L,d}^{\{J\}}$ do not depend on the fuelling state k. The symbol '$=_S$' indicates an equation that holds under the assumption of the superposition principle. We can also read it as denoting approximate equality. In general, superposition is believed to be a good approximation.

The second approximation applies to critical channel powers. Although in principle the critical channel powers also depend on both the perturbation J and the fuelling state k, in practice the fuelling state has very little influence and it is generally believed that $CCP_i^{p(J,k)}$ can be well approximated by a term $CCP_i^{\{J\}}$ that depends only on J. Thus

$$CCP_i^{p(J,k)} \approx CCP_i^{\{J\}} \ . \tag{16}$$

The superscript $\{J\}$ indicates that these quantities apply under any reactor conditions when perturbation J has arisen.

2.9 Notes

There are various reasons why the NOP equations given above differ from those that are found in the source documents I have read.

1. The focus of NOP theory in those sources was on deriving the trip setpoint TSP, whereas in this project we focus on computing CPR^{trip} given a proposed TSP.

2. The NOP equations in those sources have always been formulated in a way that assumes both the superposition principle and the CCP approximation. The reason for this is essentially pragmatic. It requires relatively little computing power to obtain computed values for $COP_i^{\{J\}}$, $FOP_{L,d}^{\{J\}}$ and $CCP_i^{\{J\}}$, because for each perturbation J only a single run of the corresponding computer code is required (based on a **time-averaged** fuelling state), whereas to obtain computed values for $COP_i^{p(J,k)}$, $FOP_{L,d}^{p(J,k)}$ and $CCP_i^{p(J,k)}$ requires one run for each combination of perturbation J and fuelling state k. So the use of these simplifications allows a huge reduction in computational requirements, but at the expense of additional uncertainty. This is represented in effect by replacing (15) with

$$COP_i^{p(J,k)} = \varepsilon_i \times COP_i^{\{J\}} \ , \qquad FOP_{L,d}^{p(J,k)} = \varepsilon_{L,d}^{*} \times FOP_{L,d}^{\{J\}} \ , \tag{17}$$

where ε_i and $\varepsilon_{L,d}^{*}$ are random error terms. Although it would be natural to also replace (16) with

$$CCP_i^{p(J,k)} = \varepsilon_i^{**} CCP_i^{\{J\}} \ , \tag{18}$$

including another random error term ε_i^{**}, the CCP approximation is regarded as sufficiently accurate to ignore this and assume $CCP_i^{p(J,k)} =$

$CCP_i^{\{J\}}$. In effect the random errors ε_i^{**} are assumed to have zero variances. In this project we allow for the possibility that the more extensive computations are feasible and hence have developed the NOP equations in the more general form, although the simplified approach with equations (17) and (18) will also be an option.

3. To facilitate the Bayesian statistical framework developed for this project, the notation is a little more complex than is generally the case with the source documents. One aspect of this is the way that superscripts distinguish between values at different time points (and so under different reactor conditions). For instance, $n(k)$ denotes a time when the reactor is operating normally in fuelling state k, whereas in the same fuelling state when normal operation is replaced by loss of regulation due to an unplanned perturbation J (but before any subsequent increase in bulk power), the time is denoted by $p(J,k)$. The superscript *trip* could have been written as $t(J,k)$ and *cal* as $c(k)$ because these are also defined relative to the fuelling state k and the perturbation J, but *trip* and *cal* are commonly used in other treatments of NOP.

3 Uncertainty

If we knew the values of all the quantities in equation (11), then we could evaluate CPR^{trip}. In particular, we could determine whether it is greater than or equal to one, and hence whether the trip occurs in time to prevent dryout. But almost all of these quantities are unknown, and so there is inevitably uncertainty regarding the performance of the trip. Uncertainty arises from different sources, and in general the quantities we are concerned with are affected by two types of uncertainty, A-uncertainty and E-uncertainty.

3.1 A-uncertainty

A-uncertainty arises from the facts that (i) a slow LOR may occur at any time in the future, and (ii) the reactor conditions vary over time. For a LOR occurring at some random point in time, the reactor will be in some random state and so CPR^{trip} is a random variable. We cannot know whether CPR^{trip} exceeds 1 for any random occurrence, but we can ask for the probability that it does. Equivalently, we can imagine reactor operation over a long period of time and ask for the **frequency** with which CPR^{trip} exceeds 1 on average.

The specific question raised by A-uncertainty is determined by the Figures of Merit (FoM) considered in the main report. For a given perturbation J, we wish to know the frequency with which $CPR^{trip} \geq 1$, so that the trip will operate in time, or more generally the frequency $Q_J(c)$ with which CPR^{trip} will exceed any specified threshold c.

The following quantities depend on the reactor conditions at the time that

a LOR arises through perturbation J, and so are subject to A-uncertainty:

$$R_i^{n(k)}, COP_i^{p(J,k)}, CCP_i^{p(J,k)}, FOP_{L,d}^{p(J,k)}, PF^{n(k)}, IRP^{cal}, CPPF^{cal}, DD_{L,d} \ . \tag{19}$$

For a slow LOR arising in random reactor conditions at a random future time, each of these quantities will have a probability distribution, its A-distribution.

3.2 E-uncertainty

E-uncertainty arises from the fact that the A-distributions are unknown. If we could observe the reactor conditions in great detail, continuously over time, for instance observing exactly the true channel powers in every channel continuously, we could evaluate these A-distributions. In reality, we have only samples of these quantities, and in most cases the sampled values are not exact, true values but computed estimates.

The SF adopts a standard statistical modelling approach in which each A-distribution is assumed to come from a suitable class of probability distributions, but the particular member of that class, identified by particular values of the parameters that define the class, is unknown. The SF also adopts a Bayesian statistical paradigm, see section 4.2, according to which the E-uncertainty regarding each A-distribution is expressed in the form of a probability distribution, its E-distribution, for the parameters. For example, a quantity may be assumed have a normal (Gaussian) A-distribution. The parameters of the normal class of distributions are the mean and variance, and so the E-uncertainty for that quantity would be expressed as an E-distribution for the mean and variance.

The E-distributions are derived wherever possible from the relevant sample data using Bayesian statistical analysis. For each group of uncertain quantities in (19), the assumed forms of A-distributions, the relevant data and the Bayesian analysis to derive E-distributions are discussed in the following section, and set out in detail in the SA, Section 7.4.

4 Statistical framework

4.1 Notation

The SF includes detailed statistical modelling for the many uncertain quantities required to compute CPR^{trip}, which in each case are formulated in terms of the logarithms of the uncertain quantities. We will denote logarithms by using bold symbols. Furthermore, many of the models are for groups of quantities that are related so that they may be correlated, in which case the symbol will represent a vector.

- $\mathbf{R}^{n(k)} = \left(\ln R_1^{n(k)}, \ldots, \ln R_{Ni}^{n(k)}\right)$, where Ni is the number of fuel channels.
- $\mathbf{COP}^{p(J,k)} = \left(\ln COP_1^{p(J,k)}, \ldots, \ln COP_{Ni}^{p(J,k)}\right)$.

- $\mathbf{CCP}^{p(J,k)} = \left(\ln CCP_1^{p(J,k)}, \ldots, \ln CCP_{Ni}^{p(J,k)}\right)$.
- $\mathbf{FOP}^{p(J,k)} = \left(\ln FOP_{1,1}^{p(J,k)}, \ln FOP_{1,2}^{p(J,k)}, \ldots, \ln FOP_{1,Nd_1}^{p(J,k)}, \ln FOP_{2,1}^{p(J,k)}, \ldots, \ln FOP_{3,Nd_3}^{p(J,k)}\right)$, where Nd_L is the number of flux detectors in logic channel L.
- $\mathbf{IRP}^{cal} = \ln IRP^{cal}$.
- $\mathbf{PF}^{n(k)} = \ln PF^{n(k)}$.
- $\mathbf{CPPF}^{cal} = \ln CPPF^{cal}$.
- $\mathbf{DD} = (\ln DD_{1,1}, \ln DD_{1,2}, \ldots, \ln DD_{1,Nd}, \ln DD_{2,1} \ldots, \ln DD_{3,Nd})$.
- $\mathbf{OP}^{p(J,k)} = \left(\mathbf{COP}^{p(J,k)}, \mathbf{FOP}^{p(J,k)}\right)$.

The same notation will be used for these quantities at other time points or reactor conditions, simply by changing the superscripts.

Data are available to inform the E-distributions for many of these quantities. The data relate to a sample of Ns actual reactor states that we denote with superscript $[s]$, for $s = 1, \ldots, Ns$. For instance $IRP^{[s]}$ is the value of IRP^{cal} in the s-th sampled state. Similarly, $COP_i^{[J,s]}$ is the channel overpower in channel i if perturbation J were to arise when the reactor was in the s-th sampled state.

Most of the data are not true values but estimates produced by computer codes such as SORO, RFSP or TUF. We denote such data using lower case symbols. For instance $r_i^{[s]}$ is the estimated ripple for channel i in the s-th sampled state. Data comprising logarithms of computed values will be denoted using the corresponding lower case bold symbols, e.g. $\mathbf{r}^{[s]} = (\ln r_1^{[s]}, \ldots, \ln r_{Ni}^{[s]})$.

4.2 Statistical methods

In the field of statistical science, several alternative philosophies or paradigms have been proposed and there is still debate over the merits of each. The most fundamental differences between different approaches can be found in the way that they handle epistemic uncertainties. To illustrate the alternatives, and to explain the choice of the Bayesian paradigm for the SF, we consider one of the simplest of all statistical problems. We have a single observation x, which is a measurement of an underlying true parameter X with measurement error ε. The error is assumed to be normally distribution with zero mean (the measurement is unbiased) and with known standard deviation y. In this example, there is aleatory uncertainty in the measurement error, because we can make many measurements and ask how often the error lies within specified bounds. The uncertainty about the true parameter X, however, is epistemic, because X will only ever have one value.

The link between this example and the SF can be seen if we suppose that x is a single log-ripple $\mathbf{r}_i^{[s]} = \ln r_i^{[s]}$ in a particular channel i for a particular data fuelling state s. Then X represents the true log-ripple $\mathbf{R}_i^{[s]} = \ln R_i^{[s]}$.[2]

[2] Strictly, the code error ε would not be aleatory in this case because it is not possible to

4.2.1 Frequentist inference

For much of the 20th century, essentially the only recognised statistical paradigm was the one we now call *frequentist*. Only aleatory uncertainty can be expressed using probabilities within the frequentist paradigm. So it is acceptable to say that ε has a normal distribution, and further that the probability distribution of x is normal with mean X and standard deviation y. In the frequentist framework, we cannot make probability statements about X, and therefore frequentist inference is somewhat convoluted. A typical frequentist inference is the confidence interval, and in this example we could say that $[x - 1.96y, x + 1.96y]$ is a 95% confidence interval for X. It is tempting to think that this means there is a 95% probability that X lies between $x - 1.96y$ and $x + 1.96y$, but this cannot be the correct interpretation because probability statements about X are not allowed in the frequentist paradigm. The correct interpretation is that if we were to obtain a long sequence of measurements $x_1, x_2, \ldots$ and computed the intervals $[x_1 - 1.96y, x_1 + 1.96y], [x_2 - 1.96y, x_2 + 1.96y], \ldots$ then X would lie in 95% of those intervals. This is a probability statement about the hypothetical sequence of future observations (which is allowed because these observations have aleatory uncertainty), not about X. It is not legitimate to infer that because 95% of intervals contain X then the probability that the specific interval $[x - 1.96y, x + 1.96y]$ contains X is 0.95 because this interval only ever has one value.

Despite these philosophical concerns, the frequentist approach is still widely adopted and defended, and its users are happy to take a confidence interval as making a useful statement about likely values of a parameter (although it is still not acceptable to infer from this an actual probability distribution for the parameter). In principle, therefore, frequentist statistical methods might be used to derive confidence intervals for $Q_J(C)$. The reason why this is not proposed in the SF is a practical one. The derivation of confidence intervals is a complex mathematical task, even for relatively simple statistical problems. Exact confidence intervals are known for some of the simplest models, and for many moderately complex problems there exist approximate confidence intervals that are widely used. But the complexity of the NOP equations and the complexity and high dimensionality of the available data mean that it would be extremely difficult to derive any frequentist inference. It is in my opinion unlikely that a suitable frequentist solution would be obtained without some years of research.

4.2.2 Fiducial inference

The fiducial approach has never been more than a curiosity, with a mere handful of adherents, but it is interesting in the present context. Although some of the details of fiducial inference are rather obscure, in effect a fiducial analysis

repeat the SORO calculations and get many different answers. Because this has the potential to cause confusion in discussion of the NOP problem I have chosen to use the terms A-uncertainty and E-uncertainty with a clear definition in the context of this problem. The A-uncertainty in the ripple $\mathbf{R}_i^{n(k)} = \ln r_i^{n(k)}$ at a future time when a LOR arises is indeed aleatory.

of the example problem would assign a probability distribution, the *fiducial* distribution, to X that is normal with mean x and standard deviation y. So the fiducial distribution inverts the sampling distribution of x (normal with mean X and standard deviation y) to produce a distribution for X. The operational meaning of a fiducial distribution is also obscure, and in practice we do not recommend fiducial inference for the SF because there is no prospect of deriving a fiducial distribution for $Q_J(C)$. However, it provides a useful reference point for the discussion of the traditional approach to the NOP problem in section 4.2.5.

4.2.3 Imprecise probabilities

An approach that has received some interest in the context of uncertainties in the predictions of complex computer codes is the P-box. This is based on an older idea of imprecise probabilities, according to which the epistemic uncertainty around a probability distribution is represented by simply saying that the distribution could be any one of some set of distributions. The P-box is a graphical representation of this where the cumulative distribution function is shown as lying within two bounding curves. The reason why this has proved to be of interest in the use of computer codes is that in principle if we have P-boxes for the inputs of the code then we could derive a P-box for the output. For the NOP problem, if P-boxes were defined for all the A-distributions then we might be able to derive a P-box for CPR^{trip} and hence a range of possible values for $Q_J(C)$ (treating it as an imprecise probability). This could be done in a Monte Carlo sampling algorithm similar to that proposed in the SF.

There are two main reasons why this has not been adopted in the SF. The first is that the interval of values would not be a very satisfactory way of expressing 'how sure we are' about $Q_J(C)$. More importantly, the theory of imprecise probabilities does not provide any practical way to derive the P-boxes for the uncertain quantities in the NOP equations, particularly since we have complex, high-dimensional data. So this is another approach that I believe is not suitable for the SF.

4.2.4 Bayesian inference

In the Bayesian paradigm, epistemic uncertainties are represented by probability distributions, encoding the analyst's knowledge and beliefs about the corresponding parameters. Such a distribution from the available data is derived using Bayes' theorem. Formally, this is the posterior distribution and Bayes' theorem requires a prior distribution that expresses what is known about the parameter before seeing the data. In the example, the analyst would assign a prior distribution for X. Supposing that this is a normal distribution with mean zero and standard deviation z, the posterior distribution of X from Bayes' theorem is normal with mean $z^2x/(y^2+z^2)$ and with standard deviation $yz/\sqrt{y^2+z^2}$. The posterior mean can be considered an estimate of X based on the data and the prior information, and we see that it lies between the prior expectation of

zero and the observation x. If the measurement x is a good one with small error, y will be small and the posterior estimate will be close to x, but if the observation is poor then the prior information becomes significant and the posterior estimate will be closer to the prior estimate of zero.

The prior distribution is potentially both a strength and a weakness for Bayesian methods. It is a strength because it allows a Bayesian analysis to incorporate additional, prior, information to supplement the information in the data. It is at least potentially a weakness because the validity of the posterior distribution depends on the prior distribution being reasonable. The Bayesian paradigm has been gaining ground in the statistical community since it emerged in the 1950s. Frequentist and Bayesian inference methods are the two dominant paradigms in statistics today, and although there are fervent advocates of each who strongly disagree with supporters of other methods, both are generally regarded as legitimate. One reason for increasing acceptance of Bayesian methods is that in complex problems they have enormous computation advantages. Powerful techniques such as Markov chain Monte Carlo (MCMC) allow Bayesian solutions to be computed for problems which defy solution by frequentist methods. This is why I have chosen the Bayesian approach for the SF.[3]

4.2.5 The traditional NOP approach

The traditional approach to deriving a trip setpoint is based on Monte Carlo sampling from probability distributions for all uncertain quantities. As such, it cannot be strictly a frequentist method because some of those uncertainties are epistemic. To consider its validity further, we can consider the example in which x and X are respectively a computed log-ripple from the SORO code and the corresponding true value. The standard deviation y is a known value representing the magnitude of computational errors in SORO-computed log-ripples. The traditional solution samples the true value X from a normal distribution with mean x and standard deviation y. As such it clearly agrees with the fiducial distribution for this case, and in general we might consider the traditional NOP analysis to be based on the principles of fiducial inference, although the traditional approach suffers from the additional difficulty that it does not separate A-uncertainties and E-uncertainties. Even if it could be adapted to a two-level Monte Carlo computation that correctly made that separation, it is not clear whether fiducial inference theory would then allow the distributions of the various NOP quantities to be propagated in such a way to yield a fiducial distribution for $Q_J(C)$.

The Bayesian approach yields a different E-distribution for the ripples. As we have seen above, the posterior distribution of X would have mean $z^2x/(y^2+z^2)$ and standard deviation $yz/\sqrt{y^2+z^2}$, both of which are less than the values x and y for the fiducial distribution. This property (which is known as *shrinkage*)

[3] I freely admit that I am one of the 'fervent advocates' for Bayesian methods, and this certainly influenced my willingness to undertake this project. But the decision to take a Bayesian approach in the SF is indeed driven by the fact that it is straightforward to apply in this problem, whereas the frequentist and other approaches are not.

derives from the prior distribution for X in the Bayesian analysis, which has a prior estimate of zero. However, this prior distribution is quite reasonable for the log-ripple $\mathbf{R}_i^{[s]}$ because if we did not have the SORO computation our best guess at any log-ripple value would indeed be zero. People do not always find shrinkage intuitively natural, but they often find the following justification more persuasive.

The essence of this argument is that if you observe a set of true values with random errors then the observations should be more variable than the true values. Formally, if the standard deviation of the true values is a and the standard deviation of the observation errors is b then the standard deviation of the observations should be $\sqrt{a^2+b^2}$. So consider the set of all SORO data for channel i, i.e. $\mathbf{r}_i^{[1]}, \ldots, \mathbf{r}_i^{[Ns]}$. Let y_r be the standard deviation of this sample. If we now follow the fiducial/traditional approach and sample the true values $\mathbf{R}_i^{[1]}, \ldots, \mathbf{R}_i^{[Ns]}$ with means $\mathbf{r}_i^{[1]}, \ldots, \mathbf{r}_i^{[Ns]}$ and standard deviations y, then the set of sampled values would be more variable than the original data, with standard deviation $\sqrt{y^2+y_r^2}$. This is counter-intuitive: the true values $\mathbf{R}_i^{[1]}, \ldots, \mathbf{R}_i^{[Ns]}$ are expected to be less variable than the SORO data, but the traditional approach makes them more variable. This is the basis for the claim in the main report that the distributions that are used for sampling in the traditional approach are not statistically sound.

In the Bayesian analysis, shrinkage means that the variability in the estimated true values is less than that of the SORO estimates. Shrinkage is natural precisely because the observations are always expected to be more variable than the underlying true values.[4]

4.3 Models

The SA, section 7.4, presents detailed statistical analyses for each group of uncertain quantities: $\mathbf{R}^{n(k)}, \mathbf{OP}^{p(J,k)}, \mathbf{CCP}^{p(J,k)}, \mathbf{IRP}^{cal}, \mathbf{PF}^{n(k)}$ and $\mathbf{DD}$. Each analysis comprises some or all of the following steps.

- A-distribution. The assumed A-distribution for the quantities is presented, and the uncertain parameters in this distribution are identified.

[4] A more complete demonstration of how the shrinkage produces the desired effect is as follows. If the prior standard deviation z is correct, so that the standard deviation of the true values $\mathbf{R}_i^{[1]}, \ldots, \mathbf{R}_i^{[Ns]}$ is z, then the standard deviation of the SORO data will be $y_r = \sqrt{z^2+y^2}$. It follows that if we now simulate the true values by sampling from their posterior distributions, the standard deviation of the simulated values will be the square root of

$$\left(\frac{z^2}{y^2+z^2}\right)^2 y_r^2 + \frac{y^2 z^2}{y^2+z^2} = \left(\frac{z^2}{y^2+z^2}\right)^2 (z^2+y^2) + \frac{y^2 z^2}{y^2+z^2} = z^2 \ ,$$

i.e. z. The shrinkage reduces the spread of estimates by exactly the right amount, so that the simulated true log-ripples have the right standard deviation, and in particular are less spread than the data. Although the Bayesian approach is dependent on the prior information being realistic, which here means specifying the prior variance z correctly, in the SF we can estimate z from the data so that the good performance of the Bayesian method is assured.

- Prior distribution. Bayesian analysis requires that information regarding parameters in addition to that deriving from the data be represented in a prior distribution.
- Data. The available data relating to these quantities are described, and a statistical model relating these to the parameters is presented.

Bayesian analysis combines the prior distribution with the data to derive the posterior distribution of the parameters. This is the required E-distribution.

For several of these groups, two or more alternative analyses are presented in the SA, and we discuss here the rationales behind the various alternatives.

4.3.1 Models for fuelling state uncertainty

Many aspects of the reactor conditions at a random future time when a perturbation arises are uncertain; for convenience we will use the term 'fuelling state' to refer to the whole set of such conditions. The random future fuelling state k creates the A-uncertainty in $\mathbf{R}^{n(k)}, \mathbf{OP}^{p(J,k)}$ and $\mathbf{CCP}^{p(J,k)}$. The A-distributions for these quantities need to reflect realistic possible values in an actual future fuelling state. For instance, the A-distribution for ripples should with a high probability generate ripples that are similar in neighbouring channels and do not deviate unrealistically far from 1 (or 0 for log-ripples). The simulation of CPR^{trip} may be sensitive to the choice of A-distribution, and so three alternatives are offered in the SF in order to explore that sensitivity and to identify the most appropriate models.

Model 1

The first model assumes that only the Ns fuelling states for which we have computed data are possible states in future. Thus, $\mathbf{R}^{n(k)} = \mathbf{R}^{[s]}$ for some $s = 1, 2, \ldots, Ns$, or $\mathbf{CCP}^{p(J,k)} = \mathbf{CCP}^{[J,s]}$ for some s. The model is clearly a simplification because in reality future fuelling states will almost certainly never exactly replicate any previous state, but the rationale for using it is two-fold. First, for large Ns it is reasonable to suppose that the computed states are a representative sample of the possible fuelling states, and therefore that this will yield a good approximation of $Q(C)$. In fact, the computed states tend to be soon after refuelling, and so may not be fully representative of the range of possible states, but if any bias is introduced in this way it is thought likely to lead to conservative computations of $Q(C)$.

The second rationale is that we know that these fuelling states, with the corresponding values of $\mathbf{R}^{n(k)}, \mathbf{OP}^{p(J,k)}$ and $\mathbf{CCP}^{p(J,k)}$, are at least possible and CNSC may be interested in performance of the proposed TSP for these particular fuelling states. By default, the A-distribution in Model 1 assumes that the Ns observed fuelling states are equally likely in future, so that each has probability Ns^{-1}, but the software inputs will allow this to be varied. In particular, it will be possible to restrict the A-distribution to a subset of these states, or even to give probability 1 to a single state. This will allow the user to explore the range of $Q_J(C)$ values if the future fuelling state were constrained to be an individual observed state.

Model 2

The remaining models allow $\mathbf{R}^{n(k)}$, $\mathbf{OP}^{p(J,k)}$ and $\mathbf{CCP}^{p(J,k)}$ to vary smoothly according to a continuous probability distribution. In Model 2 these distributions are assumed to be multivariate normal. The rationale in this case is that the multivariate normal assumption is not unrealistic, and in general normal distributions allow for statistical analyses that can either be solved analytically or at least computed simply and efficiently. The family of multivariate normal distributions is reasonably flexible, and in general the unknown parameters that are the focus of the E-uncertainty are the mean vector and variance matrix. In fact $\mathbf{R}^{n(k)}$ and $\mathbf{OP}^{p(J,k)}$ have zero mean vectors because zero is the expected value for a log-ripple or log-overpower for a random future time point. The observed fuelling states are assumed to be randomly drawn from the same population, so that for instance the $\mathbf{R}^{[s]}$ are assumed to be sampled from the same multivariate normal distribution as a future $\mathbf{R}^{n(k)}$, and this allows the parameters to be estimated, and E-distributions derived, using Bayesian analysis of the sample data.

The SA specifies that the contractors should check whether the assumptions of multivariate normal A-distributions are reasonable. It is possible that Model 2 loses important structural features in the way that ripples, overpowers or critical channel powers vary in normal operation. Model 1 should retain any such structure but the assumption of a finite discretisation of the A-distribution is likely to be too coarse, even with a large Ns, because of the high dimensionality of the parameter groups. So Model 3 introduces a nonparametric A-distribution.

Model 3

The final model relaxes the assumption of a multivariate normal A-distribution by allowing the distribution to be an arbitrary mixture of multivariate normal distributions. Such a distribution is termed *nonparametric*, meaning that it allows the true A-distribution to take any form, without restriction. The parameters are now the number of components in the mixture, the weights for the components, and the mean vectors and variance matrices for the components. This is a much more complex model, and the analysis to compute the E-distribution of all these parameters will require advanced Bayesian computational techniques. It is primarily for this reason that I have recommended in the main report that if Phase 2 of this work is commissioned the contracting team should include expertise in this kind of computation. Within Canada, I am aware of three academic groups with the necessary expertise: at UBC (Jim Zidek, Will Welch and colleagues), SFU (Derek Bingham and colleagues) and McGill (David Stephens and colleagues).

The SF gives this Model 3 explicitly just for $\mathbf{R}^{n(k)}$ but the contractors will be expected to develop analogous nonparametric models for $\mathbf{OP}^{p(J,k)}$ and $\mathbf{CCP}^{p(J,k)}$ if appropriate. Furthermore, the multivariate normal mixture model is only one of many possible nonparametric formulations, and it will be open to the contractors to propose their own preferred alternative.

4.3.2 Models using approximations

A different kind of alternative model is offered by the use of the superposition principle and/or the CCP approximation. For any given perturbation J, $\mathbf{OP}^{p(J,k)}$ depends on the fuelling state k, and the A-uncertainty is modelled using appropriate versions of Models 1, 2 and 3. However, with the superposition principle we have equation (17) which expresses $\mathbf{OP}^{p(J,k)}$ in terms of overpowers $\mathbf{OP}^{\{J\}}$ that relate to a time-averaged fuelling state and so are independent of k, plus a random error term. Similarly, (18) expresses $\mathbf{CCP}^{p(J,k)}$ in terms of critical channel powers $\mathbf{CCP}^{\{J\}}$ that are independent of k, plus another random error term.

Model 0

We therefore have a Model 0 for these cases. There is no A-distribution for $\mathbf{OP}^{\{J\}}$ or $\mathbf{CCP}^{\{J\}}$ because these quantities do not vary with time and the fuelling state. There is, however, E-uncertainty deriving from the fact that the corresponding data $\mathbf{op}^{\{J\}}$ and $\mathbf{ccp}^{\{J\}}$ are outputs of computer codes. There is A-uncertainty in the error terms because these will vary with the fuelling state, but when we use these approximations there are no data with which to estimate unknown parameters in their A-distributions, so we simply assume multivariate normal distributions with zero means and known (small) variances. There is therefore no E-uncertainty regarding the errors.

4.3.3 CPPF models

The discussion hitherto has not identified $CPPF^{cal}$ as an uncertain quantity in its own right because in principle it may be determined by equation (6) and the definition of MR^{cal} as a maximum ripple. So its A-distribution is then derived from that of $\mathbf{R}^{n(k)}$. This is identified in the SF as one of two possible models for $CPPF^{cal}$. However, another possible model is prompted by the fact that the data will include values of $MR^{[s]}$ for the computed datasets. It is therefore possible to model these and MR^{cal} as being drawn from an A-distribution of maximum ripples whose parameters can be estimated from these data, so this is given as a second model for $CPPF^{cal}$ in the SF. In view of the possibility of the simulated ripples being sensitive to the model assumed for ripples, it may be that modelling the maximum ripple directly will be more robust that deriving it from modelling the ripples. The contractors are asked to consider both models and examine whether one is more credible than the other.

4.4 Variance matrices

The outputs of computer codes SORO, RFSP and TUF are inevitably subject to computational errors, and it is necessary to specify the accuracies of each set of computations using a variance matrix. In the SA, section 5.3, optional ways are offered for the user to specify each variance matrix.

Exchangeable form

The simplest option corresponds with the way that code uncertainties are usually specified in NOP computations, for instance in the SIMBRASS code. The inputs for the exchangeable form comprise a common error variance v_c and a specific error variance v_s. If, for instance, the variance matrix relates to the errors in a set of SORO computations of ripples, SIMBRASS would refer to v_c as the channel common error and to v_s as the channel specific error.

The variance matrix is then constructed from these two variance components: all the diagonal elements of the matrix are set equal to $v_c + v_s$, while the off-diagonal elements are all set to v_c. In the example of SORO computation, the error variance in each channel is $v_c + v_s$ and the correlation coefficient between each pair of channels is $v_c/(v_c + v_s)$.

Partitioned exchangeable form

When specifying the variance matrix for errors in the RFSP computation of overpowers, the simple exchangeable form is unlikely to be appropriate because the output vectors $\mathbf{op}^{[J,s]}$ or $\mathbf{op}^{\{J\}}$ are made up of channel overpowers and flux overpowers. The inputs for the partially exchangeable form comprise five variance components – common and specific variances v_c and v_s for the channel overpowers, common and specific variances w_c and w_s for the flux overpowers, and an overall common variance v_o.

The variance matrix is constructed from these five components as follows. For the channel overpowers, the diagonal elements are $v_o + v_c + v_s$ and the off-diagonal elements are $v_o + v_c$. For the flux overpowers, the diagonal elements are $v_0 + w_c + w_s$ and the off-diagonal elements are $v_o + w_c$. Finally, all the remaining off-diagonal elements that define covariances between channel overpowers and flux overpowers are v_o. In current usage, for instance in SIMBRASS, there is no overall common error variance v_o, and this situation can be reproduced simply by inputting $v_o = 0$.

Spatial form

The exchangeable and partitioned exchangeable forms take no account of the spatial organisation of the fuel channels in the reactor, but this spatial structure is certainly a feature of the computing codes and thereby of the errors in those codes. The inputs to the spatial form are a variance v and a correlation length ℓ. The spatial form also draws on a distance matrix D whose (i, i') element $d_{i,i'}$ is the distance between channels i and i' (and hence the diagonal elements are all zero).

The variance matrix then has (i, i') element $v \exp(-(d_{i,i'}/\ell)^2)$. Each individual channel error variance is v and the correlation between errors in two channels decreases as the distance between them increases.

Partitioned spatial form

For the overpowers, we again adapt the basic form to allow for differences between channel overpowers and flux overpowers. The inputs to the partitioned spatial form are three variance components v_c, w_c and v_o, together with a correlation length ℓ.

The variance matrix for channel overpowers then has (i, i') element $(v_o +$

$v_c)\exp(-(d_{i,i'}/\ell)^2)$, that for flux overpowers has (i,i') element $(v_o+w_c)\exp(-(d_{i,i'}/\ell)^2)$, while the remaining off-diagonal elements are $v_o\exp(-(d_{i,i'}/\ell)^2)$.

General form

Finally, it is always open for the user to specify a variance matrix completely generally. The inputs for the general form comprise all the individual elements of the matrix.

The options allow the user considerable flexibility to explore sensitivity of $Q_J(C)$ to the assumed variances of code errors (and other quantities), and also to adapt to improving knowledge. Current practice, as for instance in SIMBRASS, can be reflected by use of exchangeable forms and current assumed values of variance components. Retaining exchangeable forms but altering the variance components allows the user to assess how robust the conclusions are with respect to the currently assumed error variances. In particular, the software should work with all code error variance components set to zero (although this is a highly unrealistic scenario). The spatial forms might be more realistic representations of the nature of code errors, but there is as yet no recognised quantification for the spatial structure of the errors. By utilising the spatial forms with a range of correlation lengths, the user can explore whether these forms would lead to appreciable changes in $Q_J(C)$, and if so this might justify a research effort to quantify the spatial error structure of the codes. The option to specify an arbitrary matrix in general form allows the user to define and explore even more complex error structures and to adapt to any new findings.

4.5 Constraints

The SF must take account of constraints in the definitions of ripples and overpowers. The equations (1), (3) and (4) imply the following constraints.

$$\sum_i R_i^{n(k)} CP_i^{ref} = \sum_i CP_i^{ref} = DRP \ , \tag{20}$$

$$\sum_i COP_i^{p(J,k)} R_i^{n(k)} CP_i^{ref} = \sum_i R_i^{n(k)} CP_i^{ref} = DRP \ .$$

These constraints hold for any fuelling state and any perturbation. They will also hold for all data (but note that the $COP_i^{\{J\}}$ values will only satisfy the second condition when combined with the time-averaged ripples). In terms of logarithms, these are nonlinear constraints, which are not straightforward to implement in the SF. However, although for instance the first constraint is that a weighted average (with weights CP_i^{ref}/DRP) of the ripples must be 1, I understand that it is implicit in the physics that the unweighted average should also always be close to 1. Furthermore, since the ripples do not vary individually far from 1 the average of the log-ripples should be close to 0. The modelling for ripples accordingly specifies that the A-distribution of $\mathbf{R}^{n(k)}$ is given by

$$\mathbf{R}^{n(k)} = a + \mathbf{S}\mathbf{R}^{n(k)} \ ,$$

where the scaled log-ripples $\mathbf{SR}^{n(k)}$ have a distribution that constrains them to sum to zero, and a is a constant added to each scaled log-ripple to adjust to the correct constraint (20), i.e.

$$a = \ln DRP - \ln \left(\sum_i \exp(\mathbf{SR}_i^{n(k)}) CP_i^{ref} \right) \ .$$

A similar approach is used in modelling the overpowers.

APPLICATION OF BAYES METHOD IN EVALUATION OF ROP/NOP TRIP SETPOINT — PHASE I
Software Appendix

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March 23, 2016

1 Background

[*Task 4.4 of this contract calls for "a work plan for implementation of the proposed framework in a computer code". This appendix provides a detailed specification of the work required to implement the Software Framework (SF). It is intended as a possible basis for a tender document for potential bidders for that work. On the other hand, this appendix will assume familiarity with the language and ideas of Bayesian statistics.*

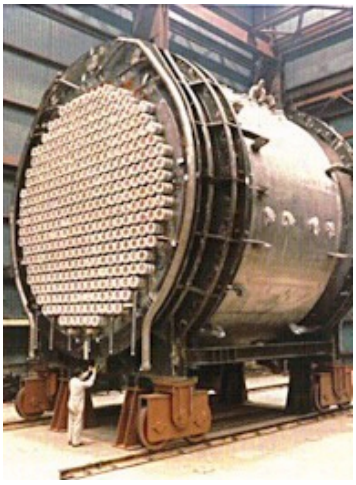
In this background section CNSC should introduce the project and the nature of the tender.]

2 Overview

This section provides a simplified description of the CANDU reactor and the NOP protection system which is the subject of this project. As such, the description is inevitably inaccurate in a number of details, but it is intended to be adequate to provide basic understanding of the problem.

2.1 CANDU reactors

Nuclear power generation in Canada is based on the CANDU reactor design. In the reactor core are a number of *fuel channels* (typically more than 300), each running the length of the core. The channels are arranged so that in cross-section they form a circular shape, as shown in the figure below.



A feature of CANDU reactors is that individual channels can be refuelled without shutting down the reactor. The power being generated by an individual channel at any time is called its *channel power*, and the sum of channel powers for all the channels in the reactor is the bulk *reactor power*.

When the reactor is designed and commissioned, a *design reactor power* (DRP) is established, together with *reference channel powers* CP_i^{ref} (for $i = 1, 2, \ldots, Ni$, where Ni is the number of fuel channels) that sum to DRP. During actual operation, the current reactor power may differ from DRP, and we refer here to the ratio of the actual reactor power at time t to DRP as the *power factor* (PF^t). We express the actual channel power in channel i at time t in the form

$$PF^t \times R_i^t \times CP_i^{ref} \ ,$$

where R_i^t is the *ripple* in channel i at time t. Ripples represent deviation of the pattern of channel powers at any given time from the pattern established in the reference channel powers. One reason for ripples is the fact that some channels have been refuelled more recently than others, and so more power may be being generated in some channels, and less in others, than would arise in the reference situation. Therefore the ripples at time t depend on the reactor state at that time, and in particular on the fuelling history up to that time. The ripples in some channels will be greater than 1 and in others they will be less than 1, because

$$\sum_i R_i^t \times CP_i^{ref} = DRP = \sum_i CP_i^{ref} \ .$$

2.2 NOP protection system

Nuclear reactors have many safety systems installed. In this project we are concerned with the *neutron overpower* (NOP) protection system, which is designed to react to a slow *loss of regulation* (LOR). The LOR is conceived in a stylised way as comprising a sequence in which, first, something happens to cause a *perturbation* of the channel powers (also known as a *flux shape*), and then the reactor power increases steadily. The perturbation is described by *channel overpowers*, such that after the perturbation has occurred the channel power in channel i becomes

$$PF^t \times COP_i^t \times R_i^t \times CP_i^{ref} \ ,$$

where the channel overpowers COP_i^t at time t depend primarily on the nature of the event causing the perturbation, but also on the reactor state before the perturbation.

Through the distortion of the channel powers, and through the subsequent rise in reactor power, the LOR may cause one or more fuel channels to experience *dry-out*, which occurs when the power in a channel exceeds its *critical channel power* (CCP). The NOP protection system is designed to prevent that occurring. It comprises a number of *flux detectors* which are installed in various locations through the core (i.e. at various points along a number of different fuel channels) and which monitor the neutron flux continuously at those points, together with a *trip setpoint* (TSP). The flux detectors are wired into a number of *logic channels* (typically 3), and if the reading in at least one detector in each logic channel exceeds the TSP the NOP protection will trip into operation, rapidly reducing power throughout the reactor.

In order to adapt to changing reactor conditions (particularly the ripples) and also to correct for drift in their responses, the detectors are *calibrated* regularly. At calibration, all the detectors are set so that they read a common value which is the product of two terms, the *channel power peaking factor* $CPPF$ and the *indicated reactor power* IRP. The $CPPF$ is the maximum value, over the channels in a central region of the core (the CPPF region), of the currently estimated ripples (but subject to a fixed minimum value $CPPFm$). The IRP is the current estimated reactor power.

2.3 Critical power ratio at trip

During LOR, the detector readings are perturbed by *flux overpowers* that reflect the nature of the perturbation, and they also increase proportionately to the subsequent rise in bulk reactor power. The NOP protection system will have succeeded in avoiding dry-out if, when the reactor power has risen to the point where the flux detector readings are high enough to trigger the trip, the channel powers are all still less than their CCP_i values. The *critical power ratio at trip*, CPR^{trip}, is the minimum over all fuel channels of the

ratio of CCP_i to the channel power at the time when the trip occurs. If $CPR^{trip} \geq 1$ then the trip operates successfully. If $CPR^{trip} < 1$ then there is the risk of dryout in at least one channel, and even lower values may lead to core damage.

The TSP is a single number that is hard-wired into the reactor, whereas the channel powers (and therefore CPR^{trip}) depend in complex ways on the reactor state and the particular perturbation associated with the LOR, so in practice the NOP protection may not always trip successfully to prevent dry-out. It is important to evaluate the performance of the TSP in terms of how often the trip will operate successfully, for each of a range of specific perturbations that might arise.

2.4 Uncertainties

The value of CPR^{trip} for any LOR event is uncertain for at least two reasons. First, the reactor state varies over time, and so the values of the many quantities that are needed to compute CPR^{trip} are uncertain. This source of uncertainty is called A-uncertainty (from the term 'aleatory'). A-uncertainty is characterised in this problem as uncertainty about the reactor state at a random future time point when the LOR arises.

If we knew the probability distributions, the A-distributions, that describe how all the various quantities vary over time, then we could in principle calculate the frequency with which $CPR^{trip} \geq 1$. However, we also have uncertainty about these distributions. For instance we do not know the A-distribution of the ripple R_i^t in channel i for a random time t. Uncertainty about the A-distributions is E-uncertainty (from the term 'epistemic') and results in uncertainty about the frequency with which $CPR^{trip} \geq 1$.

2.5 Statistical framework

In Phase 1 of this project, a Bayesian statistical framework for this problem has been developed, in which the E-uncertainty is characterised as posterior distributions, derived from available data, of the uncertain parameters in the A-distributions. In Phase 2 of the project, this Bayesian framework will be implemented in software.

3 Tasks

[*This section should set out CNSC's list of Tasks, milestones and deliverables for Phase 2.*]

4 Notation

4.1 Subscripts and superscripts

Subscripts on a symbol distinguish between values of the corresponding quantity at different locations in the reactor.

- Subscript i denotes fuel channel i, and ranges from 1 to Ni.
- Subscript L, d denotes flux detector d in logic channel L, where L ranges from 1 to NL and d from 1 to Nd_L.

Superscripts distinguish between values of the corresponding quantity in different reactor states.

- Superscript $n(k)$ denotes the reactor operating normally in fuelling state k at a random future instant before a perturbation.
- Superscript $p(J, k)$ denotes the reactor state after perturbation J has occurred in fuelling state k at a random future instant without any increase in bulk power, where J ranges from 1 to NJ.
- Superscript $\{J\}$ denotes the reactor state after perturbation J has occurred in a time-averaged fuelling state.

- Superscript $[s]$ denotes the reactor state when computation set s was produced, where s ranges from 1 to Ns.
- Superscript $[J, s]$ denotes the reactor state that would have arisen if perturbation J had occurred when computation set s was produced.
- Superscript *trip* denotes the reactor state when the NOP trip is triggered.
- Superscript *cal* denotes the reactor state when the flux detectors were last calibrated before a perturbation occurs.

For example, $R_i^{n(k)}$ is the ripple in channel i when the reactor is operating normally in fuelling state k, while $FOP_{L,d}^{\{J\}}$ is the flux overpower at flux detector d in logic channel L after perturbation J has occurred in a time-averaged fuelling state.

4.2 Symbol style

Symbols in bold face are the natural logarithms of the same symbols in italic face. For example, $\mathbf{R}_i^{n(k)} = \ln R_i^{n(k)}$.

Symbols in lower case are computed estimates of the same symbols in upper case. For example, $fop_{L,d}^{\{J\}}$ is a computed estimate of $FOP_{L,d}^{\{J\}}$. (Symbols in mixed case are quantities that are not estimated by nuclear physics computer codes.)

4.3 Vectors

A symbol which has no subscript, when it is also used with a subscript, denotes a vector comprising all subscripted values of that symbol. For example, $\mathbf{R}^{n(k)} = (\mathbf{R}_1^{n(k)}, \mathbf{R}_2^{n(k)}, \ldots, \mathbf{R}_{Ni}^{n(k)})$.

Where the subscript is usually L, d, the vector is composed with subscript d varying first. For example, $\mathbf{FOP}^{\{J\}} = (\mathbf{FOP}_{1,1}^{\{J\}}, \mathbf{FOP}_{1,2}^{\{J\}}, \ldots, \mathbf{FOP}_{1,Nd_1}^{\{J\}}, \mathbf{FOP}_{2,1}^{\{J\}}, \ldots\ldots, \mathbf{FOP}_{NL,Nd_{NL}}^{\{J\}})$.

4.4 Combined overpowers

A symbol OP with any superscript (and in any style) denotes a vector concatenating the corresponding COP and FOP vectors, For example, $\mathbf{OP}^{\{J\}} = (\mathbf{COP}^{\{J\}}, \mathbf{FOP}^{\{J\}})$. The number of elements in a combined overpowers vector is denoted by $No = Ni + \sum_L Nd_L$.

5 Input

The tables in this section give the symbols and names for all the quantities that may be required as input to the software. The third column in each table specifies whether this input is always required or whether it is required depending on the value of some other input.

The final column, 'Validation', specifies any constraints on the inputs that should be checked when validating the input. In this column, the following notation is used.

- First, if the input is a vector or a matrix, the dimensions are shown followed by a multiplication sign. For example, "$5\times$" denotes a vector of 5 values, while "$(Ni, Ni)\times$" denotes a matrix of Ni rows and columns.
- If no dimensions are shown, the input is a scalar.
- This is followed by an indication of the type of values: R denotes real values, I denotes integer values, and the permitted range is shown with the symbol $\in$. For example, "$5 \times I \in [0, 3]$" denotes an input vector of 5 elements that must all be integers in the range 0 to 3.

- If no permitted range is shown, the input is required only to be non-negative. For example, "R" denotes a non-negative real-valued scalar.
- Where appropriate, this may be followed by other constraints. For example, "$(Ni, Ni) \times R$, symmetric" denotes a real symmetric matrix of Ni rows and columns.

5.1 Basic inputs

Symbol	Name	When required	Validation
TSP	Trip setpoint	Always	R
Ni	Number of fuel channels	Always	I
NL	Number of logic channels	Always	I
Nd	Number of flux detectors per logic channel	Always	$NL \times I$
NJ	Number of perturbations	Always	I
Ns	Number of observed fuelling states	Always	I
Nc	Number of critical values for CPR^{trip}	Always	I
Nt	Number of threshold values for $Q(C)$	Always	I
C	Critical values for CPR^{trip}	Always	$Nc \times R$
T	Threshold values for $Q(C)$	Always	$Nt \times R \in (0, 1)$
p	Relative frequencies of perturbations	Always	$NJ \times R \in [0, 1]$, $\sum_J p_J = 1$
DRP	Design reactor power	Always	R
CP^{ref}	Reference channel powers	Always	$NI \times R$, $\sum_i CP_i^{ref} = DRP$
$CPPFm$	Minimum channel power peaking factor	Always	R
CPm	Maximum channel power	Always	R

Some of these inputs are implicit in the values or constraints of others, and could in principle be omitted from the inputs. For instance, Ni, Nd, NJ, Ns, Nc and Nt might be omitted and inferred from the dimensions of other inputs, and DRP might be inferred from the constraint on CP^{ref}. However, including them explicitly as inputs will allow validation checks on inputs that would otherwise not be available.

5.2 Switches

Symbol	Name	When required	Validation
Mr	$\mathbf{SR}^{n(k)}$ model number	Always	$I \in [1, 3]$, $Mr = 1$ if $Mop = 1$ or $Mccp = 1$
Mf	$CPPF^{cal}$ model number	Always	$I \in [0, 1]$
Mop	$\mathbf{SOP}^{p(J,k)}$ model number	Always	$I \in [0, 2]$, $Mop \neq 2$ if $Mr = 1$
$Mccp$	$\mathbf{CCP}^{p(J,k)}$ model number	Always	$I \in [0, 2]$, $Mccp \neq 2$ if $Mr = 1$
Mw	Switch for $[s]$ weights	Always	$I \in [0, 1]$
Sum	Output switch for $Q(C)$ summaries	Always	$I \in [0, 3]$

5.3 Variances

A number of inputs that are required under the different models are variance matrices. Five alternative input forms are available for such matrices. We introduce the following notation:

- I denotes an identity matrix;
- 1 denotes a unit matrix (all of whose elements are 1).

Dimensions of I and 1 are as required for conformability. In particular, 1 may be a vector.

1. **Exchangeable form:** For a variance matrix V of dimensions (Ni, Ni), the input comprises v_c and v_s (validation: $2 \times R$). The matrix is $V = v_s I + v_c 1$.
2. **Partitioned exchangeable form:** For a variance matrix V of dimensions (No, No), the input comprises v_c, v_s, w_c, w_s and v_0 (validation: $5 \times R$). The matrix is

$$V = \begin{pmatrix} v_s I + (v_c + v_0)1 & v_0 1 \\ v_0 1 & w_s I + (w_c + v_0)1 \end{pmatrix},$$

where the upper-left partition is of dimensions (Ni, Ni).
3. **Spatial form:** For a variance matrix V of dimensions (Ni, Ni), the input comprises v and ℓ (validation: $2 \times R$). The matrix is $V = v \exp(-(D_{11}/\ell)^2)$, where the distance matrix D is an input of dimensions (No, No), partitioned as $D = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix}$ with D_{11} of dimensions (Ni, Ni), and where the division by ℓ, the square and the exponential are applied element-wise.
4. **Partitioned spatial form:** For a variance matrix V of dimensions (No, No), the input comprises v_c, w_c, v_0 and ℓ (validation: $4 \times R$). The matrix is

$$V = \begin{pmatrix} (v_c + v_o)\exp(-(D_{11}/\ell)^2) & v_o \exp(-(D_{12}/\ell)^2) \\ v_o \exp(-(D_{21}/\ell)^2) & (w_c + v_o)\exp(-(D_{22}/\ell)^2) \end{pmatrix} .$$

5. **General form:** A variance matrix of dimensions (N, N) may always be input explicitly (validation $(N, N) \times R$, positive-definite, symmetric).

In addition to two scalar variances Vpf and Vdd, the following table specifies five variance matrices, the switch inputs that define their input form, and the possible values of those switches. The validation entry for the matrix itself is empty, because the validation depends on the input form (and is specified with each form above).

Symbol	Name	When required	Validation
Vpf	Bulk power error variance	Always	R
Vdd	Detector drift variance	Always	R
$Ssoro$	Switch for $Vsoro$	Always	$I \in \{1, 3, 5\}$
$Vsoro$	SORO error matrix	Always	
$Srfsp$	Switch for $Vrfsp$	Always	$I \in \{2, 4, 5\}$
$Vrfsp$	RFSP error matrix	Always	
$Stuf$	Switch for $Vtuf$	Always	$I \in \{1, 3, 5\}$
$Vtuf$	TUF error matrix	Always	
$Ssop$	Switch for $Zsop$	$Mop = 0$	$I \in \{2, 4, 5\}$
$Zsop$	Superposition error matrix	$Mop = 0$	
$Sccp$	Switch for $Zccp$	$Mccp = 0$	$I \in \{1, 3, 5\}$
$Zccp$	CCP approx error matrix	$Mccp = 0$	

5.4 Miscellaneous inputs

These inputs are not always required

Symbol	Name	When required	Validation
Hi	CPPF region	$Mf = 0$	$Ni \times I \in [0, 1]$
D	Distance map	(*)	$(No, No) \times R$, (**)
w	$[s]$ weights	$Mw = 1$	$Ns \times R \in [0, 1]$, $\sum_s w_s = 1$

(*) D is required in case of any of the following: $Ssoro = 3$, $Srfsp = 4$, $Stuf = 3$, $Ssop = 4$, $Sccp = 3$.

(**) D must be a valid distance matrix. In particular, it must be symmetric, all diagonal elements must be zero and for every triplet i, i', i'' the triangle inequality $d_{i,i''} \leq d_{i,i'} + d_{i',i''}$ must hold. This can be validated by checking that $\exp(-D)$ is symmetric and positive definite, with all diagonal elements equal to 1.

5.5 Data

Finally, the computed data inputs are given in the following table. An additional column 5 says how many instances of each item are required. A value Ns indicates that these data are required for each of the Ns computed fuelling states, and a value NJ indicates that they are required for each of the NJ perturbations.

Symbol	Name	When required	Validation	Instances
$r^{[s]}$	Ripples	Always	$Ni \times R$	Ns
$op^{[J,s]}$	Overpowers	$Mop \neq 0$	$No \times R$	$Ns \times NJ$
$op^{\{J\}}$	Superposition overpowers	$Mop = 0$	$No \times R$	NJ
$ccp^{[J,s]}$	Critical channel powers	$Mccp \neq 0$	$Ni \times R$	$Ns \times NJ$
$ccp^{\{J\}}$	Approx crit channel powers	$Mccp = 0$	$Ni \times R$	NJ
$IRP^{[s]}$	Indicated reactor power	Always	R	Ns
$MR^{[s]}$	Maximum CPPF ripple	$Mf = 1$	R	Ns

It may be useful for the input Hi to be provided always, rather than just when $Mf = 0$ as specified in the previous section. Then it would be possible to add a validation check that $MR^{[s]} = \max_i(Hi_i \times r_i^{[s]})$.

5.6 Input media

[*This section should specify any requirements that CNSC has for how inputs should be provided to the software. Graphical user interface? File-based input to allow batch processing? GUI plus data file? File formats.*]

6 Outputs

6.1 Required outputs

The required outputs are

- $P_J(C_c, T_t)$ for $J = 1, \ldots, NJ$, $c = 1, \ldots, Nc$ and $t = 1, \ldots, Nt$.
- $P(C_c, T_t)$ for $c = 1, \ldots, Nc$ and $t = 1, \ldots, Nt$.

6.2 Optional outputs

If $Sum = 1$ or 3, then additional outputs should be provided characterising the E-distribution of $Q_J(C_c)$. For each $J = 1, \ldots, NJ$ and $c = 1, \ldots, Nc$, this will comprise the median, quartiles and extreme values of $Q_J(C_c)$ obtained in the outer loop from the sampling in the inner loop of the two-level Monte Carlo algorithm in section 7.2 below. A natural way to display them would be as box-and-whisker plots.

If $Sum = 2$ or 3, then a separate output file should be created containing all the sampled values of $Q_J(C_c)$ for each J and c.

6.3 Output media

[*This section should specify any requirements that CNSC has for how outputs should be provided by the software. File formats, display formats, output direct to Excel?*]

7 Algorithms

In implementing these algorithms, care should be taken over the distinction between bold and roman symbols. In many cases, the algorithms require logarithms of input values, for instance TSP is an input but its logarithm **TSP** is used in the equations below.

7.1 Critical power ratio at trip

The fundamental equation for the NOP calculation is

$$\mathbf{CPR}^{trip} = \min_i \left(\mathbf{CCP}_i^{p(J,k)} - \mathbf{PF}^{n(k)} - \mathbf{COP}_i^{p(J,k)} - \mathbf{R}_i^{n(k)} - \mathbf{CP}_i^{ref} \right) - \mathbf{F}^{trip} \ ,$$

where

$$\mathbf{F}^{trip} = \mathbf{TSP} - \min_L \max_d \left(\mathbf{DD}_{L,d} + \mathbf{FOP}_{L,d}^{p(J,k)} \right) - \mathbf{CPPF}^{cal} - \mathbf{IRP}^{cal} \ .$$

However, if $\mathbf{F}^{trip} < 0$, then $\mathbf{CPR}^{trip}$ should instead be computed as

$$\mathbf{CPR}^{trip} = \min_i \left[\mathbf{CCP}_i^{p(J,k)} - \mathbf{PF}^{n(k)} - \left(E^{trip} \times \mathbf{COP}_i^{p(J,k)} \right) - \mathbf{R}_i^{n(k)} - \mathbf{CP}_i^{ref} \right] ,$$

where E^{trip} is the solution to the equation

$$\mathbf{TSP} = \min_L \max_d \left[\mathbf{DD}_{L,d} + \left(E^{trip} \times \mathbf{FOP}_{L,d}^{p(J,k)} \right) \right] + \mathbf{CPPF}^{cal} + \mathbf{IRP}^{cal}$$

7.2 Two-level Monte Carlo

The required outputs $P(C_c, T_t)$ and $P_J(C_c, T_t)$ are computed by a two-level Monte Carlo algorithm.

1. Repeat for $J = 1, 2, \ldots, NJ$ {
   Set counter $p(c,t) = 0$ $(c = 1, \ldots, Nc;\ t = 1, \ldots, Nt)$
   Repeat $M1$ times (**outer loop**) {
   Sample parameters from their E-distributions
   Set counter $q(c) = 0$ $(c = 1, \ldots, Nc)$
   Repeat $M2$ times (**inner loop**) {
   Sample uncertain quantities from their A-distributions
   Compute $\mathbf{CPR}^{trip}$
   Increment $q(c)$ if $\mathbf{CPR}^{trip} \geq \mathbf{C}_c$ $(c = 1, \ldots, Nc)$ }
   Set $Q_J(C_c) = q(c)/M2$ $(c = 1, \ldots, Nc)$
   Increment $p(c,t)$ if $Q_J(C_c) \geq T_t$ $(c = 1, \ldots, Nc;\ t = 1, \ldots, Nt)$ }
   Output $P_J(C_c, T_t) = p(c,t)/M1$ $(c = 1, \ldots, Nc;\ t = 1, \ldots, Nt)$ }
2. Output $P(C_c, T_t) = \sum_J p_J P_J(C_c, T_t)$ $(c = 1, \ldots, Nc;\ t = 1, \ldots, Nt)$

The simulation sizes $M1$ and $M2$ should be large enough to produce stable Monte Carlo estimates of the probabilities $P_J(C_c, T_t) = \Pr(Q_J(C_c) \geq T_t)$, and the contractors should justify the way in which these values are chosen (either adaptively or as preset values based on test cases). Notice that in the inner loop the A-distributions are fixed using the values of parameters that have been sampled in the outer loop.

If $Sum \neq 0$, the algorithm should be adapted to collect all the $M1$ sampled values of $Q_J(C)$ during the outer loop, so that the median, quartiles, maximum and minimum may be computed and reported if $Sum = 1$ or 3, and so that the complete set of sampled values may be output if $Sum = 2$ or 3.

7.3 Scaled ripples and overpowers

The ripples must satisfy

$$\sum_i R_i^{n(k)} \times CP_i^{ref} = DRP$$

and the channel overpowers must satisfy

$$\sum_i COP_i^{p(J,k)} \times R_i^{n(k)} \times CP_i^{ref} = DRP \ .$$

In the Bayesian modelling, we work with *scaled* log-ripples and log-overpowers which sum to zero. Thus, $\mathbf{SR}^{n(k)}$ and $\mathbf{SOP}^{p(J,k)}$ are respectively scaled log-ripples corresponding to $\mathbf{R}^{n(k)}$ and scaled log-overpowers corresponding to $\mathbf{OP}^{p(J,k)}$. We similarly have scaled quantities $\mathbf{SR}^{[s]}$ and $\mathbf{SOP}^{[J,s]}$ as the scaled log-ripples and log-overpowers for computation set s. The computed estimates $\mathbf{sr}^{[s]}$ and $\mathbf{sop}^{[J,s]}$ are derived by scaling

the input data. Formally, we introduce the matrix K of dimensions (Ni, Ni) and the matrix M of dimensions (No, No) defined as

$$K = I - Ni^{-1}1 \ ,$$
$$M = \begin{pmatrix} K & 0 \\ -Ni^{-1}1 & I \end{pmatrix} \ ,$$

and then the scalings are

$$\mathbf{sr}^{[s]} = K.\mathbf{r}^{[s]} \ , \qquad \mathbf{sop}^{[J,s]} = M.\mathbf{op}^{[J,s]} \ .$$

The dot here denotes matrix multiplication. In the two-level Monte Carlo algorithm, $\mathbf{SR}^{n(k)}$ and $\mathbf{SOP}^{p(J,k)}$ are sampled from their A-distributions, then the sampled values of $\mathbf{R}^{n(k)}$ and $\mathbf{OP}^{p(J,k)}$ are computed from

$$\mathbf{R}^{n(k)} = \mathbf{SR}^{n(k)} + \left(\mathbf{DRP} - \ln(\textstyle\sum_i SR_i^{n(k)} CP_i^{ref})\right) 1 \ ,$$
$$\mathbf{OP}^{p(J,k)} = \mathbf{SOP}^{p(J,k)} + \left(\mathbf{DRP} - \ln(\textstyle\sum_i SCOP_i^{p(J,k)} R_i^{n(k)} CP_i^{ref})\right) 1 \ .$$

Notice that these equations require $SR_i^{n(k)}, R_i^{n(k)}$ and $SCOP_i^{p(J,k)}$, which are the exponentials of the sampled $\mathbf{SR}_i^{n(k)}, \mathbf{R}_i^{n(k)}$ and $\mathbf{SOP}_i^{p(J,k)}$, together with $\mathbf{DRP} = \ln DRP$

In the Bayesian modelling, the constraint that the scaled log-ripples and log-channel overpowers must sum to zero is implemented by variance matrices that imply that condition. In particular, the input variance matrices $Vsoro$ and $Vrfsp$ apply to the computed log-ripples and log-overpowers, so that the implied variance matrices for their scaled counterparts are

$$Vsoro^* = K.Vsoro.K \ ,$$
$$Vrfsp^* = M.Vrfsp.M^T \ .$$

Such matrices are singular, so care should be taken in implementing the Bayesian analyses and in sampling from the A-distributions.

7.4 Bayesian modelling

The following table sets out all the Bayesian models for the various uncertain quantities needed to compute CPR^{trip}.

The first column lists the uncertain quantities and, where appropriate, the alternative models.

In the two-level Monte Carlo algorithm, the parameters are sampled in the inner loop from their A-distributions as specified in the second column.

Except for $\mathbf{CPPF}^{cal}$ with model $Mf = 0$, $\mathbf{PF}^{n(k)}$ and $\mathbf{DD}$, these A-distributions contain unknown parameters, which are listed in the third column.

Values are assigned to these unknown parameters in the outer loop, by sampling from their E-distributions. The E-distributions are derived using Bayesian analysis from the prior distribution in column 4 and the data model in column 5. The E-distribution is the posterior distribution from this analysis.

In some cases the prior distributions and data models are sufficiently simple for the posterior E-distribution to be derived analytically, so that there is an explicit E-distribution to sample from. In the remaining cases, it will be necessary to use Markov chain Monte Carlo (MCMC) methods to generate a random sample from the posterior E-distribution. Such samples are ready-made for use in the inner loop. In the case of $\mathbf{SR}^{n(k)}$ with model $Mr = 3$, the computation is likely to require reversible jump MCMC.

The contractors must provide full details of the Bayesian methods used. Where the E-distribution can be derived analytically, the solution should be shown. Where MCMC methods are used, the contractors are expected to obtain a suitably efficient sampling scheme, which should be documented fully. Choices of burn-in and thinning should be specified, with reasons for the values chosen or the algorithm used to determine them automatically.

The final column of the table points to numbered notes following the table.

Model	A-distribution	Parameters	Prior	Data	Notes
$\mathbf{SR}^{n(k)}$					
$Mr=1$	$\mathbf{SR}^{n(k)} = \mathbf{SR}^{[S]}$, $\Pr(S=s)=w_s$ $(s=1,\ldots,Ns)$	$\mathbf{SR}^{[s]}$ $(s=1,\ldots,Ns)$	$\mathbf{SR}^{[s]} \sim N(0,Wsoro)$ indep	$\mathbf{sr}^{[s]} \sim N(\mathbf{SR}^{[s]},Vsoro^*)$ indep	(1) (2)
$Mr=2$	$\mathbf{SR}^{n(k)} \sim N(0,Wsoro)$	$Wsoro$	Noninformative	$\mathbf{sr}^{[s]} \sim N(0,Vsoro^*+Wsoro)$ indep	(3) (4)
$Mr=3$	$\mathbf{SR}^{n(k)} \sim N(m_G, b\times Wsoro)$, $\Pr(G=g)=a_g$ $(g=1,2,\ldots,Ng)$	Ng, a_g $(g=1,\ldots,Ng, \sum_g a_g=1)$, $b\in(0,1)$, m_g $(g=1,\ldots,Ng)$, $Wsoro$	$Ng \sim Ge(0.25)$, a_g uniform, b uniform, $m_g \sim N(0,(1-b)\times Wsoro)$ indep, $Wsoro$ noninformative	$\mathbf{sr}^{[s]} \sim N(\mathbf{SR}^{[s]},Vsoro^*)$ indep	(3) (5)
$\mathbf{CPPF}^{cal}$					
$Mf=0$	$\mathbf{CPPF}^{cal} = \max(\mathbf{CPPFm}, \max_i(Hi_i\times \mathbf{r}_i^{n(k)}))$				(6)
$Mf=1$	$\mathbf{CPPF}^{cal} = \max(\mathbf{CPPFm}, \mathbf{MR}^{n(k)})$, $\mathbf{MR}^{n(k)} \sim N(mf,vf)$	mf,vf	Noninformative	$\mathbf{MR}^{[s]} \sim N(mf,vf)$ indep	
$\mathbf{SOP}^{p(J,k)}$					
$Mop=0$	$\mathbf{SOP}^{p(J,k)} \sim N(\mathbf{SOP}^{\{J\}},Zsop)$	$\mathbf{SOP}^{\{J\}}$	$\mathbf{SOP}^{\{J\}} \sim N(0,Wsp)$	$\mathbf{sop}^{\{J\}} \sim N(\mathbf{SOP}^{\{J\}},Vrfsp^*)$	(7)
$Mop=1$	$\mathbf{SOP}^{p(J,k)} = \mathbf{SOP}^{[J,S]}$	$\mathbf{SOP}^{[J,s]}$ $(s=1,\ldots,Ns)$	$\mathbf{SOP}^{[J,s]} \sim N(0,Wrfsp)$ indep	$\mathbf{sop}^{[J,s]} \sim N(\mathbf{SOP}^{[J,s]},Vrfsp^*)$ indep	(2′) (8)
$Mop=2$	$\mathbf{SOP}^{p(J,k)} \sim N(0,Wrfsp)$	$Wrfsp$	Noninformative	$\mathbf{sop}^{[J,s]} \sim N(0,Vrfsp^*+Wrfsp)$ indep	(3′) (4′)
$\mathbf{CCP}^{p(J,k)}$					
$Mccp=0$	$\mathbf{CCP}^{p(J,k)} \sim N(\mathbf{CCP}^{\{J\}},Zccp)$	$\mathbf{CCP}^{\{J\}}$	$\mathbf{CCP}^{\{J\}} \sim N(mca,Wca)$	$\mathbf{ccp}^{\{J\}} \sim N(\mathbf{CCP}^{\{J\}},Vtuf)$	(7′)
$Mccp=1$	$\mathbf{CCP}^{p(J,k)} = \mathbf{CCP}^{[J,S]}$	$\mathbf{CCP}^{[J,s]}$ $(s=1,\ldots,Ns)$	$\mathbf{CCP}^{[J,s]} \sim N(mtuf,Wtuf)$ indep	$\mathbf{ccp}^{[J,s]} \sim N(\mathbf{CCP}^{[J,s]},Vtuf)$ indep	(2″) (8)
$Mccp=2$	$\mathbf{CCP}^{p(J,k)} \sim N(mtuf,Wtuf)$	$mtuf,Wtuf$	Noninformative	$\mathbf{ccp}^{[J,s]} \sim N(mtuf,Vtuf+Wtuf)$ indep	(4″)
$\mathbf{IRP}^{cal}$	$\mathbf{IRP}^{cal} \sim N(mi,vi)$	mi,vi	Noninformative	$\mathbf{IRP}^{[s]} \sim N(mi,vi)$ indep	
$\mathbf{PF}^{n(k)}$	$\mathbf{PF}^{n(k)} \sim N(\mathbf{IRP}^{cal},Vpf)$				(9)
$\mathbf{DD}$	$\mathbf{DD} \sim N(0,Vdd\times I)$				(9)

Notes

(1) $w_s = Ns^{-1}$ if $Mw=0$.

(2) $Wsoro$ is set equal to the empirical Bayes estimate $Wsoro = Ns^{-1}\sum_s \mathbf{sr}^{[s]}\mathbf{sr}^{[s]T} - Vsoro^*$.

(2′) $Wrfsp$ is set equal to the empirical Bayes estimate $Wrfsp = Ns^{-1}\sum_s \mathbf{sop}^{[J,s]}\mathbf{sop}^{[J,s]T} - Vrfsp^*$.

(2″) $mtuf$ and $Wtuf$ are set equal to the empirical Bayes estimates $mtuf = Ns^{-1}\sum_s \mathbf{ccp}^{[J,s]}$ and $Wtuf = Ns^{-1}\sum_s \left(\mathbf{ccp}^{[J,s]} - mtuf\right)\left(\mathbf{ccp}^{[J,s]} - mtuf\right)^T - Vtuf$.

(3) The noninformative prior distribution for $Wsoro$ should be constrained so that rows and columns sum to zero. The posterior E-distribution will then have the same property.

(3′) The noninformative prior distribution for $Wrfsp$ should be constrained so that its first Ni rows and columns sum to zero. The posterior E-distribution will then have the same property.

(4) The contractors should explore whether the assumption of a multivariate normal distribution for $\mathbf{sr}^{[s]}$ is reasonable. This can be done both by looking at the $\mathbf{sr}^{[s]}$s and by comparing results with Model 3.

(4′) The contractors should explore whether the assumption of a multivariate normal distribution for $\mathbf{sop}^{[J,s]}$ is reasonable for a given J, and should propose a nonparametric Model 3 if it does not seem realistic.

(4″) The contractors should explore whether the assumption of a multivariate normal distribution for $\mathbf{ccp}^{[J,s]}$ is reasonable for a given J, and should propose a nonparametric Model 3 if it does not seem realistic.

(5) The geometric distribution is designed to give high probability to Ng being small. Alternative values to 0.25 for the parameter could be considered.

(6) In this model, the A-distribution of $\mathbf{CPPF}^{cal}$ is induced by that of $\mathbf{SR}^{n(k)}$, with the hypothetical SORO-computed ripples $\mathbf{r}^{n(k)}$ being derived by applying the transformation of section 7.3 to $\mathbf{sr}^{n(k)} \sim N(\mathbf{SR}^{n(k)}, Vsoro^*)$. No Bayesian analysis or E-distribution is needed. The contractors should consider whether it is better to model the ripples and derive the CPPF as in this model, or to model CPPF directly as in model 1.

(7) Wsp is set equal to the empirical Bayes estimate $Wsp = NJ^{-1}\sum_J \mathbf{sop}^{\{J\}}\mathbf{sop}^{\{J\}T} - Vrfsp^*$. Note that within the two-level Monte Carlo algorithm, each J is handled separately, and almost all other computations use only the current value of J, and can be done within the J loop. This computation, however, must be done before the Monte Carlo algorithm begins.

(7′) mca and Wca are set equal to the empirical Bayes estimates $mca = Ns^{-1}\sum_J \mathbf{ccp}^{\{J\}}$ and $Wca = NJ^{-1}\sum_J \left(\mathbf{ccp}^{\{J\}} - mca\right)\left(\mathbf{ccp}^{\{J\}} - mca\right)^T - Vtuf$. Note that within the two-level Monte Carlo algorithm, each J is handled separately, and almost all other computations use only the current value of J, and can be done within the J loop. These computations, however, must be done before the Monte Carlo algorithm begins.

(8) In the A-distribution, S takes the same value as in the A-distribution of $\mathbf{SR}^{n(k)}$.

(9) There are no uncertain parameters in this A-distribution.

8 Testing

8.1 Face validity

Like any major software development, the NOP trip setpoint assessment software must be thoroughly tested and quality assured. One important set of tests concern face validity, i.e. ensuring that as the inputs change in particular ways the outputs change as expected.

The following examples of face validity tests should be implemented, but others may also be proposed.

- If the quantity of input data (SORO and/or RFSP datasets) is reduced, the overall E-uncertainty should increase, and hence the E-uncertainty in CPR^{trip} should increase.
- If the specified variances of code errors are increased, then again the E-uncertainty in CPR^{trip} should increase.
- If the $\mathbf{ccp}^{\{J\}}$ data are reduced, for instance by 2%, then the E-distribution of CPR^{trip} should shift towards lower values, and in particular $P_J(C,T)$ should decrease.
- Perturbations that have uniformly smaller effects on channel powers should lead to generally higher values of CPR^{trip}, and in particular should have higher $P_J(C,T)$.

8.2 Benchmarking

Another tool for validation is benchmark testing. Test examples are constructed where the correct solution is known, and the software solution is checked to see if it is close enough to the correct solution to be fit

for purpose. In this problem there are no real data for which true values are known, so it is necessary to fabricate "realistic" scenarios.

For instance:

- The datasets of SORO, RFSP and TUF data are assumed to be true values for a sample of fuelling states. The software is first run in **SR**, **SOP** and **CCP** model 1 and **CPPF** model 0 using these "true" values with no E-uncertainty (e.g. zero code error variances) to obtain the true $Q_J(C)$ values. Then new data are generated by taking these "true" values and adding random code errors consistent with the known accuracies of the codes. The software is run with several new datasets generated in this way, to check that the outputs are consistent with the true $Q(C)$s.
- Values of parameters for the multivariate normal A-distributions of **SR**, **SOP** and **CCP** model 2 are assumed (for instance the posterior means of these parameters from an earlier run of the software). The software is adapted to fix these values for a single ($M1 = 1$) pass through the outer loop of the Monte Carlo algorithm in order to obtain true $Q_J(C)$ values. The software is then run with several random data sets generated according to the corresponding data models, to check that the outputs are consistent with the true $Q_J(C)$s.

8.3 Reserved data

Another useful validation test uses held-back data. The software is run with a reduced set of data, for instance two-thirds of the original number of fuelling state samples, to predict the values of the remaining, held-back, data. This requires the inner loop to be modified so that instead of computing CPR^{trip} it simulates datasets from additional random fuelling states. The inner loop normally simulates true vales, but it is now modified to add random code errors to simulate computed data. In this way a sample of computed datasets from their predictive distribution is generated, and this is compared with the held-back data.

8.4 Log file

A file containing a log for diagnostic purposes should be created in each run of the software. At least the following data should be logged.

- Date, time, settings of input switches
- Diagnostics for convergence of MCMC algorithms
- If, at any time in the inner loop of the two-level Monte Carlo algorithm, the value of $\mathbf{F}^{trip}$ is negative, so that the second formula for $\mathbf{CPR}^{trip}$ is used in section 7.1, then record the value of J.
- If, at any time in the inner loop of the two-level Monte Carlo algorithm, the value of $\mathbf{PF}^{n(k)} + \mathbf{COP}_i^{p(J,k)} + \mathbf{CP}_i^{ref}$ exceeds **CPm**, then record its value and the channel number i. If $Mr = 1$, then also record the value of S. Such occurrences may indicate a problem with the **SR** model, because normal reactor operation tries to manage the core so that no channel exceeds this value.

9 Tender requirements

[*This section should set out formal CNSC requirements for tenders.*]

CNSC research project R612.1:
"Application of Bayes method in evaluation of ROP/NOP trip setpoint - Phase I"

Disposition of Comments from Bruce Power and Ontario Power Generation on the Draft Report

Anthony O'Hagan

March, 2016

I am pleased to note that the fundamental components of the report, namely the NOP equations, the two-level Monte Carlo simulation and the Bayesian modelling, have been accepted with no critical comments.

High Level Comments from Ontario Power Generation

(E-mail, Z. Catovic (OPG) to D. Serghiuta (CNSC), E-DOCS# 4964887, March 18, 2016)

	Comment	**Disposition**
1	The proposed approach distinguishes between "effectiveness" and "risk" problem. The effectiveness problem addresses individual perturbation and the risk problem is then considered as a weighted summation over all possible perturbations. A similar approach was taken in EVS-2006. The ITP has noted that this may lead to incorrect results, in particular, when there are significant variations among the probabilities for the various perturbations. The ITP report should be reviewed to consider the relevance of the finding to the proposed approach.	The software allows different values to be input for the probabilities of different perturbations, so that the robustness of the weighted sum can be explored.

2	The proposal includes a discussion of Testing & Benchmarking. More details would be helpful in understanding if the benchmarking approach will be similar to that applied to the EVS-NOP, to allow for direct comparisons.	The benchmarking is necessarily rather different from that applied to EVS. First, EVS was addressing a different question – the choice of a trip setpoint rather than the assessment of a given trip setpoint. Second, because the underlying mathematics of EVS was generic it was possible to create very simple benchmark tests that bore no relation to the NOP problem but were still valid tests of that underlying theory. This is not possible for the Bayesian SF, which is tailored very specifically to the NOP context.
3	Appendix A describes a scenario where trip may occur during perturbation, i.e., cases where trip may occur before the flux shape becomes fully established, and a scaling parameter and a slightly different formulation of critical power ratio. A more detailed review will be appropriate to confirm the equation addresses the problem as posed.	I doubt if it is possible to observe a perturbation becoming established at sufficient resolution and with sufficient accuracy to verify or refute any proposed model for the process.
4	Appendix A provides a discussion of uncertainty. It appears from the discussion that the A-uncertainty for CCP is only associated with fuelling state. While fuelling state is a key parameter defining the neutron physics of the reactor, the CCP at any given time also depends on thermal hydraulic conditions, which are not dependent on fuelling state, but are also subject to A-uncertainty due to random variations in the process conditions, i.e., flow, temperature and pressure. It is not clear if this variation is included in the A-uncertainty model.	**Change made.** There was no intention to limit the sources of A-uncertainty.
5	The report introduces the concept of the true value, but it is not clear how E-uncertainty is defined with respect to the true value. In particular, it is proposed in the report to sample the E-uncertainty as part of the outer loop in a two-level Monte-Carlo simulation. It is not clear how the distribution derived with the proposed technique is related to the true value.	E-uncertainty relates to the true values of *parameters* in assumed models for A-uncertainty. The statistical modelling is specifically designed to derive the E-distributions as posterior distributions from Bayesian analysis of data. It is these distributions that are sampled in the outer loop.

High Level Comments from Bruce Power

(E-mail, O. Nainer (BP) to D. Serghiuta (CNSC), E-DOCS# 4964894, March 16, 2016)

	Comment	**Disposition**
Page 9	"Bayesian SF" could have been abbreviated further	This section is quoting CNSC instructions for this project. I think it is better not to change those by abbreviating further.
Page 9	Not clear what decision making is here. Could it be what is acceptable for a functional failure probability and its uncertainty?	This section is quoting CNSC instructions for this project. I have interpreted the instruction as saying that the SF should provide information to support regulatory decision-making, without indicating or suggesting any preferred decision.
Page 10	The NOP analysis used to be called ROP (Regional Overpower Protection). This is to make the point that a bulk power increase is not necessarily needed to reach the dryout condition ie in conditions of high flux tilts dryout can also happen. Again, here and bullet above, bulk power increase is not required. NOP protects against local distortions as well.	Indeed, the question of whether the NOP trip setpoint protects against such a situation is considered in section 2.7 of the Technical Appendix as the case of "trip during perturbation".
Page 11	To me unknown is exaggerated in this context. Not known precisely is probably better term. In principle we have enough fine granularity on the flux shapes, ripples, etc.	**Change made.** Although to a statistician, "unknown" does not have the strength implied by the word in common usage, I agree that weaker wording would be better in this context.
Page 11	In my opinion this work could add more value if it would attempt to go into areas less explored ie propagate uncertainties and general treatment of errors for cases that are more realistic ie dynamic transients. I understand this was not in the scope provided for this work, however multiple arguments have been provided with respect to the risks associated to the slow LOR and ultimately supporting the current EVS approach.	Although not practical currently, it is true that explicit dynamic transient simulation may be available in the future. However, this would be likely to involve different Bayesian modelling that cannot be anticipated at this time.

Page 11	Time element is not an issue in the slow LOR. It is assumed the transient occurs very slowly. In real life the neutronic transient are relatively fast and are over in at about 10 seconds.	"Just in time" refers to the trip functioning just before the bulk power increase reaches the level where dry-out may occur. However fast or slow the bulk power increase occurs in practice, this point is well defined.
Page 11	The traditional methodology was actually building a distribution of TSP and the acceptance probability was applied to select the lower bound.	**Change made.** The text should distinguish between the traditional methodology's original focus and the focus of this project.
Page 15	I do have a problem with the approach; I understand that this is typical Bayes application and I understood this in the PRA application where failure rates are postulated and then corrected based on experience. In the NOP case there are already uncertainty distributions for most if not all uncertainties hence starting the process from assumptions seems to be a step backward.	Where there is knowledge about the error processes, then of course the assumptions should match that knowledge. However, the uncertainty distributions that are generally used are not strictly *known* to be correct. They are based on samples of actual measurements, and if those samples are extensive we may apply calculations in which for instance an error is normally distributed with a given variance, but the distribution is not *known* to be normal and the variance is not *known* to take that value. The use of the normal distribution with the given variance is an assumption. In fact, where the error is modelled as a multiplicative term 1+e, where e is given a normal distribution, then this assumption is actually *known* to be false because the normal distribution allows the theoretical possibility that 1+e is negative (no matter how small the variance), which is unphysical. The SF instead assumes a normal additive error on the log scale, which is theoretically more realistic. With small error variance, the sample data would not be able to distinguish between the two uses of the normal distribution.
Page 15	I would also assume that the solution of the error propagation is similar to EVS or come synthetic cases can be constructed. Different uncertainties propagations will render different results and those would have nothing to do with Bayes application.	It is very difficult to say how the error propagation relates to EVS, since in EVS there is no explicit inference about underlying parameters.
Page 16	With this being said, what is the classification of the EVS - B or F ?	EVS is frequentist. The text here says that to develop a frequentist solution would require a substantial research programme, with no assurance of success. EVS was indeed the result of a substantial

		research programme, and in my opinion was not a success. It was designed to choose a trip setpoint, and to produce a frequentist method for assessing the effectiveness of a given trip setpoint would be a fresh problem.
Page 16	CPR at trip is in effect ratio of margin-to-dryout to margin-to-trip. may worth mentioning.	I appreciate that this is a standard way to describe the CPR at trip, but I find it potentially confusing because it is supposed to hold at any time during the bulk power increase and depends on the assumption of uniform power increase. Defining it as the margin to dry-out at the moment when the margin to trip has reduced to 1 seems to me to be more definite and a true reflection of "at trip".
Page 18	There are more parameters that vary in time and affect the dryout. it may be worth mentioning that the list is not comprehensive or perhaps include all A error sources. for example reactor inlet hear temperature, reactor outlet header pressure.	**Change made.**
Page 19	It may be useful to provide some guidance on data when modelling. For example in the super position if the errors made ignoring the actual initial burnup distribution are reasonably smaller than the perturbation itself, then this E-uncertainty could be rolled in with the others.	Yes, this becomes part of the superposition error variance Zsop in the Software Appendix (when Mop = 0).
Page 19	This would be the superposition principle that works for both channels and detectors.	Yes, the word "overpowers" includes both channel overpowers and flux overpowers.
Page 19	weakly informative - how are the prior distribution derived or what assumptions are made ?	I use the term "weakly informative" to indicate that the prior distribution is designed to convey only the genuinely available prior information and nothing more. Details of what this means for each set of parameters are given in the Software Appendix.
Page 20	I think from a SF perspective this is fine. One lesson we learned from EVS was that one should consider how the parameters sampled from the E-distribution affect CPRattrip and this may not be trivial.	For any given set of parameter values sampled from E-distributions in the outer loop, there is a probability distribution for CPR^{trip} generated by A-uncertainty in the inner loop. Whilst in principle one could output all of these M1 x M2 CPR^{trip} values, this would be an enormous amount of data and it is not clear how they could

		usefully be analysed.
Page 21	This seems to be the first reference to cppf. if cppf was explicit in cprattrip (and would be treated as a constant) then this face validity criteria would be evident.	In the Technical Appendix, CPPF is explicit in the formula for CPR^{trip}. However, it depends on the fuelling state and is subject to measurement error and is therefore not a constant. The effect of increasing the *minimum* CPPF (which is indeed a constant) on CPR^{trip} is thus not deterministic but stochastic.
Page 21	This is somewhat intuitive but difficult to put in practice and largely arguable. also it depends if the perturbation is favourable to detector or not.	**Change made.** Thank you for causing me to look again at this bullet point. On reflection, I find it unsound.
Page 25	As I pointed in the meeting, there is another condition that is met by the SORO CPs. ie ;max(CPi)<CPmax for all ripples.	This has been implemented in the Software Appendix using the new input CPm. It is not practical to enforce this condition because it depends on both ripples and reactor power, but the section on creating a log file requires instances where the condition is not met during the simulation to be recorded. This will serve as a diagnostic, and may trigger revision of the software.
Page 26	The wording implies that the two parameters ie DRP and CPref are derived together. This is not the case DRP is derived from nuclear design and CPref from NOP optimizations studies; CPref can be reoptimized	I take the point but feel that the text is adequate as it stands. For the purpose of this report, it is only necessary that both are fixed and known.
Page 26	I think that jumping from dryout to sheath failure is quite a long way. Actually there have been studies discussing that operation beyond dryout with a limited high temperature for a limited time is acceptable (ie fuel sheath is maintained).	The text only says that dry-out “may lead” to rupture.
Page 27	This is margin to dryout.	Yes, but it is also CPR.
Page 28	This is essentially margin to trip.	Specifically, it is the margin to trip as computed at the point when the perturbation is established but no bulk power increase has yet ensued. The point about margin to trip and margin to dry-out is that they change during bulk power increase, and this is something that I found confusing in standard treatments of NOP. That is largely why I have tended to prefer other terminology.
Page 29	I find the introduction of E unnecessary, artificial and inconsistent with the rest of the NOP methodology. Let's	I have introduced this precisely because of the situation that you raise in your comments on page 10. Concerning the change in

	recall that the assumption we have is a linear power change after the perturbation is established, while here the exponent is applied during the perturbation effectively changing it. The relationship works fine in absence of this factor as the initial power level is rather irrelevant.	channel powers after the perturbation is established, i.e. during bulk power increase, the assumption is that if one channel power has increased by a factor F then all have increased by the same factor. This is a natural assumption in the context of increasing bulk reactor power. As far as I can see, there is no natural assumption for how a perturbation becomes established, nor am I aware of any data or studies relating to this. Indeed, I doubt if such studies are practicable. In the light of this, I have chosen to make a *convenient* assumption. I doubt if it would be materially different from any other plausible assumption in practice. The software log file will record all instances when this formula is invoked.
Page 51	Similar to the comment in the main body: The calculation of CPRtrip may not be straightforward (as the section above would seem to suggest) once the uncertainties are factored in; especially if the true value is not used . This in itself can be the future source of much disagreement with EVS, hence it may deserve some more discussion.	Within the inner loop of the algorithm *true* values of all quantities are simulated by drawing from their A-distributions. Given all these true values, the computation of CPR^{trip} simply involves applying the formulae. It is indeed straightforward. The Bayesian algorithm involves none of the contortions required by EVS.
Page 51	As per my other comment in the app.a the soro cps also reflect the compliance activities ie CPSORO<CPmax.	See my response to your comment on page 25.
Page 55	I suggest that some thought is put into constructing some benchmarks to test against EVS (or synthetic cases based on EVS).	A benchmark should be an accepted correct answer against which a software can be tested. EVS does not provide a benchmark in this sense.